

# Homework

Miklós Abért

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**Exercise 1** Show that if  $\lim_{n \rightarrow \infty} a_n = 0$  and  $(b_n)$  is bounded, then

$$\lim_{n \rightarrow \infty} (a_n b_n) = 0.$$

**Exercise 2** Prove that the sequence  $a_n = 2^{-n}$  converges to 0.

**Exercise 3** Show that if  $s > 1$  and  $n$  is a natural number, we have

$$s^n \geq 1 + n(s - 1)$$

**Exercise 4** Prove that for each real number  $t$  with  $0 < t < 1$ , the sequence  $x_n = t^n$  converges to zero. (Hint: use  $s = t^{-1}$ .)

**Exercise 5** Show that for each real number  $|t| < 1$ , the sequence

$$a_n = 1 + t + t^2 + \dots + t^{n-1}$$

satisfies

$$\lim_{n \rightarrow \infty} a_n = 1/(1 - t).$$

**Exercise 6** Prove that for any real number  $a$ , the sequence  $x_n = a^n/(n!)$ , where  $n! = 1 \cdot 2 \cdot \dots \cdot n$ , converges to zero.

**Exercise 7** Let

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

and

$$q(x) = b_m x^m + b_{m-1} x^{m-1} + \dots + b_1 x + b_0$$

be polynomials. What is

$$\lim_{n \rightarrow \infty} \frac{p(n)}{q(n)} ?$$

**Exercise 8** What is

$$\lim_{n \rightarrow \infty} \sqrt[n]{n} ?$$