

Homework

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Exercise 1 Show that if

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

is a polynomial, such that n is odd and $a_n \neq 0$ then there exists $c \in \mathbb{R}$ with $p(c) = 0$.

Exercise 2 Show that if $a, b \geq 0$ then

$$\sqrt{ab} \leq \frac{a+b}{2}$$

and equality holds if and only if $a = b$.

Exercise 3 Show that if $a, b \in \mathbb{R}$ then

$$\frac{a+b}{2} \leq \sqrt{\frac{a^2+b^2}{2}}$$

and equality holds if and only if $a = b$.

Exercise 4 Show that if $a, b \geq 0$ then

$$\frac{2}{\frac{1}{a} + \frac{1}{b}} \leq \sqrt{ab}$$

and equality holds if and only if $a = b$.

Exercise 5 Show that if $a, b, c \in \mathbb{R}$ then

$$\frac{a+b+c}{3} \leq \sqrt{\frac{a^2+b^2+c^2}{3}}$$

and equality holds if and only if $a = b = c$.

Exercise 6 Is there a real function $f : \mathbb{R} \rightarrow \mathbb{R}$ that takes on every real number an even number of times?