

# Homework

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due January 22

Just to make sure:

$$[0, 1] = \{x \in \mathbb{R} \mid 0 \leq x \leq 1\}$$

$$[0, 1) = \{x \in \mathbb{R} \mid 0 \leq x < 1\}$$

$$(0, 1) = \{x \in \mathbb{R} \mid 0 < x < 1\}$$

**Exercise 1** Find a bijection between  $(0, 1)$  and  $\mathbb{R}$  (write one up precisely).

**Exercise 2** Sheet 4, Exercise 13.

Hint: can you do it if its standing (that is, the speed vector is zero)?

**Exercise 3** Sheet 4, Exercise 20.

For some reason, I like this problem a lot.

And the next one, too.

**Exercise 4** Let  $S$  be an infinite set of natural numbers of the form  $2^a 3^b$  ( $a, b$  are natural numbers). Show that there exists  $s, s' \in S$  such that  $s \neq s'$  and  $s$  divides  $s'$ .

Many of you must know this one, but it will help a lot with sets later.

**Exercise 5** There is an army inspection and the general finds that the soldiers are unshaven. Under the threat of execution, the barber is ordered to shave exactly those in the army who do not shave themselves. He commits suicide. Why?

**Exercise 6** Let  $X$  be an uncountable subset of  $\mathbb{R}$ . Then  $X$  has a limit point.