Sheet 29: π is irrational

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Below is a sketch of the proof that π is irrational.

Suppose indirectly that

$$\pi = \frac{a}{b}$$

For a natural n let

$$f(x) = \frac{1}{n!}x^n(a - bx)^n$$

and let

$$F(x) = f(x) + \ldots + (-1)^j f^{(2j)}(x) + \ldots + (-1)^n f^{(2n)}(x)$$

1. $f(x) = f(\pi - x)$
2. $0 \leq f(x) \leq \frac{1}{n!}\pi^n a^n$ $(0 \leq x \leq \pi)$
3. $f^{(j)}(0)$ and $f^{(j)}(\pi)$ are integers for all j
4. $F(0)$ and $F(1)$ are integers
5. $F + F'' = f$
6. $(F' \sin - F \cos)' = f \sin$
7. $\int_0^\pi f \sin$ is an integer
8. This leads to a contradiction if n is large enough