HW on wiggly functions: Math 131 section 10, Fall 2007

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Justify all your answers clearly and concisely. Make sure you let the grader know which question you're answering at every point. There aren't as many questions as it looks: there's not many different ideas here, and there are 14 questions because I'm breaking the task of graphing the function down into tiny steps.

1. Graph $f(x) = 1 - x + \lfloor x \rfloor$, between -2 and 2. *Hint: You already know how to graph $x - \lfloor x \rfloor$, and in fact $f(x) = 1 - (x - \lfloor x \rfloor)$.*

2. Calculate $\lim_{x \to 2^+} f(x)$, using the limit definition. Then calculate $\lim_{x \to 2^-} f(x)$, using the limit definition.

3. We're going to define a new function $s(x)$, where the s stands for *sawtooth*, by

$$s(x) = \begin{cases} x - \lfloor x \rfloor & \text{if } \lfloor x \rfloor \text{ is even} \\ 1 - x + \lfloor x \rfloor & \text{if } \lfloor x \rfloor \text{ is odd} \end{cases}$$

   What is $s(1)$? What is $s(n)$ if n is an odd integer? What is $s(2)$? What is $s(n)$ if n is an even integer?

4. What is $s(n + 1/2)$, if n is an integer? What is $s(n + 1/4)$, if n is an integer? *Hint: one of these depends on n.*

5. Show that s is an even function, i.e. for all x, $s(x) = s(-x)$.

6. Calculate $\lim_{x \to 0} s(x)$ *Hint: use left and right limits to help you.* Similarly, calculate $\lim_{x \to -1} s(x)$.

7. Graph $s(x)$ for values of x between -3 and 3.

8. Remember how to compose functions? Let $w(x) = s(1/x)$, where w stands for *wiggly*. What is the natural domain of w?

9. What is $w(1)$? What is $w(1/n)$ if n is a positive integer? *Hint: that depends on n again.*

10. Graph $w(x)$ for values of x between $1/6$ and 2. *Hint: You'll need to use nice big axes, and there are no straight lines in this graph!*

11. Show that w is an even function. Graph $w(x)$ for values of x between -2 and $-1/6$, on the same axes as the previous question.

12. What do you think happens to w near zero? Use the limit definition to show that $\lim_{x \to 0} w(x) \neq 0$. Then, try to show that for all L, $\lim_{x \to 0} w(x) \neq L$. In other words, that the limit of $w(x)$ at $x = 0$ is not defined.

13. Let $t(x) = x \cdot w(x)$. Graph $t(x)$ for x between -1 and 1. For once, you can be less than 100% precise, although you still need to label your axes and so on.

14. Try to use the squeeze theorem to show that $\lim_{x \to 0} t(x) = 0$