Exercise 1 Suppose $H$ is a Hilbert space with a countable basis \{$u_1, u_2, \ldots$\}. Define $\Lambda : H \to H$ to be the linear function with

$$\Lambda(u_n) = n^{-1} u_n, \quad n = 1, 2, \ldots$$

• Show that $\Lambda$ is bounded and find $\|\Lambda\|$.

• Let $U = \{v \in H : \|v\| < 1\}$. Show that $\Lambda U$ does not contain $\delta U$ for any $\delta > 0$.

• Why does this not contradict the open mapping theorem?