

**Math 312, Autumn 2007**

**Problem Set 5**

Rudin, Chapter 5: 4, 5, 6, 8, 9, 11, 15, 18, 22

Probability Notes: Exercise 5.7, 5.8

**Exercise 1** *Suppose  $H$  is a Hilbert space with a countable basis  $\{u_1, u_2, \dots\}$ . Define  $\Lambda : H \rightarrow H$  to be the linear function with*

$$\Lambda(u_n) = n^{-1}u_n, \quad n = 1, 2, \dots$$

- *Show that  $\Lambda$  is bounded and find  $\|\Lambda\|$ .*
- *Let  $U = \{v \in H : \|v\| < 1\}$ . Show that  $\Lambda U$  does not contain  $\delta U$  for any  $\delta > 0$ .*
- *Why does this not contradict the open mapping theorem?*