Math 312, Autumn 2008
Problem Set 5

Reading: Rudin Chapter 5 (through p. 108)
Probability Notes, Section 5

Rudin, Chapter 5: 2, 6, 8, 9, 10, 11, 15, 18, 19, 22
Probability Notes: Exercise 5.7, 5.8

Exercise 1 *Suppose H is a Hilbert space with a countable basis $\{u_1, u_2, \ldots\}$. Let $\Lambda : H \to H$ be a continuous linear function with*

$$\Lambda(u_n) = n^{-1} u_n, \quad n = 1, 2, \ldots. \tag{1}$$

- *Explain why there exists a unique continuous linear Λ satisfiying (1) and find $\|\Lambda\|$.*

- *Let $U = \{v \in H : \|v\| < 1\}$. Show that ΛU does not contain δU for any $\delta > 0$.*

- *Why does this not contradict the open mapping theorem?*