

Math 312, Autumn 2009
Problem Set 5

REMINDER: This assignment is due by Wednesday, Nov. 4 at 11:30 am even though there is no class that week. You may bring it to class or put it directly in C. Carstea's mailbox.

Reading: Rudin Chapter 5 (through p. 108)
Probability Notes, Section 5

Rudin, Chapter 5: 3, 4, 8, 9, 11, 16, 17, 19
Probability Notes: Exercise 5.7, 5.8

Exercise 1 *Suppose H is a Hilbert space with a countable basis $\{u_1, u_2, \dots\}$. Let $\Lambda : H \rightarrow H$ be a continuous linear function with*

$$\Lambda(u_n) = n^{-1} u_n, \quad n = 1, 2, \dots \quad (1)$$

- *Explain why there exists a unique continuous linear Λ satisfying (1) and find $\|\Lambda\|$.*
- *Let $U = \{v \in H : \|v\| < 1\}$. Show that ΛU does not contain δU for any $\delta > 0$.*
- *Why does this not contradict the open mapping theorem?*