ABSTRACTS: 2005 SUMMER VIGRE REU

All concepts mentioned in the abstracts will be carefully defined

DISCRETE MATHEMATICS

Weeks 1–8: Laci Babai

First Module: Weeks 1–4

Groups, Graphs, Matrices, Avalanches, and More: The Abelian Sandpile Model

Originating in statistical physics and nearly simultaneously and independently introduced in algebraic graph theory and in theoretical computer science in the 1990s, the Abelian Sandpile Model associates a variety of structures with a diffusion process on finite graphs. The model gives rise to a remarkably rich theory which connects the fields of graph theory, stochastic processes, commutative semigroups and groups, matrices and determinants, lattices in n-dimensional space, algorithms, number theory, fractals, and more. In the context of the Abelian Sandpile Model we shall learn about cyclotomic polynomials, the Jordan-Holder Theorem, the Laplacian and the matrix-tree theorem, and a lot more.

Open problems abound, covering all areas mentioned - you can pick your favorite.

This is a fun topic. We shall learn most prerequisites along the way.

Second Module: Weeks 5–8

Potpourri

The course covers topics in number theory, geometry, combinatorial structures, linear algebra, and discrete probability, highlighting surprising interactions between these areas. Students will discover the field through solving sequences of challenging problems. A number of open problems will also be discussed. The overlap with recent years will be minimal.

PQ: (For both). Basic linear algebra and discrete math, or consent of instructor.

Students interested in either module are strongly encouraged to take Math-28400, a.k.a. CS-27400, Honors Combinatorics and Probability, offered in Spring, see http://www.cs.uchicago.edu/courses/descriptions.php. That course is offered in alternate years only.

You are also encouraged to talk to the instructor at the beginning of the Spring quarter, even if you do not take Combinatorics and Probability.

SPECIAL LECTURES in ANALYSIS

Weeks 1 and 2: Robert Fefferman

Maximal Functions in Analysis

This will be a self-contained introduction to the theory of maximal functions, which are some of the most important objects in modern harmonic analysis and partial differential equations. We shall consider various generalizations of the Fundamental Theorem of Calculus, and wind up with an elementary introduction to Calderon-Zygmund Theory. One of the basic ingredients of the study of maximal functions is a deep understanding of the geometry of collections of simple sets such as balls or rectangles in Euclidean spaces. We shall investigate this geometry in some detail.

PROBLEMS, COMBINATORICS, AND *A PIACERE*

Weeks 1–4: Miklos Abert

Personal

'Problem, somewhat neglected but still beautiful, seeks extroverted math enthusiast to spend some quality time together. Playfully competitive is a plus.'

This year I am organizing a problem-solving course. Problems range from fun exercises through beauty contest winners to honest research starters. Topics will include real analysis, topology, combinatorics, algebra and number theory. Students of all levels are welcome. Minicourses will be offered for those who are in need of some background.

Weeks 5–6: Carley Klivans

Algebraic Combinatorics - Hyperplane Arrangements

This two week course will be a taste of algebraic combinatorics through the study of arrangements of hyperplanes. Topics may include intersection lattices, questions of enumeration, and connections to graphs and matroids. In this pursuit, we will consider some basic structures and techniques of algebraic combinatorics such as generating functions, characteristic polynomials, the theory of posets and the mobius algebra. (1) You do not need to know what any of the words above mean to take this class, although a knowledge of linear algebra would be quite helpful. (2) We will be able to quickly look at a number of interesting problems ranging from the very accessible to the unsolved.

Weeks 7–8: Peter May

A Piacere

That is Italian for "at your pleasure", meaning that I can't make up my mind. I will give topics in algebra or in topology, or both. If algebra, I will among other topics introduce the foundations of algebraic geometry, offering several elementary intuitive proofs of Hilbert's Nullstellensatz. If topology, I will introduce fixed point theory, including the Nielson number, which is a fundamental topic in algebraic topology that is rarely taught in courses in algebraic topology.

GEOMETRY

Weeks 1–4: Benson Farb and Chris Hruska (2 weeks each)

Mapping Class Groups

The mapping class group of a surface is the group of self-homeomorphisms modulo homotopy (=continuous deformation). The study of these groups lies at the intersection of geometric/combinatorial group theory, low-dimensional topology, complex analysis, and algebraic geometry. There has been a recent renaissance in this area.

This course will be an introduction to the basics of the theory. The course will be taught at a number of levels at once, and will hopefully be interesting from the freshmen through graduate student level. A number of accessible open questions will be presented.

Selected topics: surface topology, Alexander trick, Dehn twists, the classical modular group, braid groups, the complex of curves, the finite generation theorem, the lantern and chain relations, Hurwitz's formula and the 84(g-1) theorem.

PQ: The basics of point-set topology and group theory might be useful, but are not essential.

Weeks 5–8: Uri Bader and Roman Muchnik (2 weeks each)

The Dynamics of Hyperbolic Manifolds

In this four week course, we will study hyperbolic geometry from a dynamical point of view. This is a central area of modern mathematics, which is accessible to undergraduate students. The course will begin with an introduction to hyperbolic geometry and an introduction to ergodic theory and dynamical systems. We will learn hyperbolic geometry by presenting a few simple models that arise in basic complex geometry. We will study the group of isometries of hyperbolic space. A special feature of this group is a mixing property, known as the Howe-Moore property. The tools that we develop will be used to study compact hyperbolic manifolds. We will see that these manifolds possess strong dynamical properties. Finally we will use these dynamical properties to gain a better understanding of the geometry.

TOPICS IN APPLIED MATHEMATICS

Weeks 1 and 2: Fausto Cattaneo

Transport of Vectors and Scalars

Consider the following experiment: take some hot water and some cold water and put them together. Is it possible to stir the resulting mixture in such a way that the hot water gets hotter and the cold water gets colder? No, it is not. The result is always lukewarm water no matter how you stir it. In fact stirring the mixture just makes matters worse in the sense that the lukewarm state is achieved sooner. Now repeat the experiment with a vector field (for example the magnetic field in an electrically conducting fluid). If you do nothing the vector field will eventually decay away. But what happens if you stir? Amazingly, if you do it the right way, the magnitude of the vector field can increase without bounds. Why is that? More to the point, how should one stir to get this unbounded growth? In this course I will address some of these issues by looking at some simple properties of the partial differential equations that describe the transport of vectors and scalars. In particular I will show how, in some cases, it is possible to derive effective equations that describe the evolution of average quantities.

Weeks 3 and 4: Peter Gordon

Introduction to Nonlinear Waves

Many phenomena in physics, chemistry, biology and other applications are nonlinear in nature and can be described by nonlinear partial differential equations, or systems of such equations. There are a number of "universal" model equations such as the Ginzburg-Landau equation, the KPP-Fisher equation, the Burgers equation and a dozen others which describe a wide variety of nonlinear effects. These equations possess a nice class of solutions called 'traveling waves'. These solutions assume a certain shape (usually localized in space) and translate with constant speed. An interesting feature of these solutions is that they often are globally attractive. That is, the solution of the Cauchy problem approaches the traveling wave as time progresses. In this short course we will discuss a number of models such as the Burgers, the KPP-Fisher, and the Sine-Gordon equations and their traveling wave solutions, as well as the stability of such solutions.

Weeks 5 and 6: Eduard Kirr

Resonances in Oscillator-Wave Systems

Oscillator-wave systems are now ubiquitous in Quantum Mechanics, Statistical Physics (Bose-Einstein Condensates) and Optics. However, their evolution is far from being completely understood.

For this module I plan to start from the mechanical experiment of two oscillators connected with a string. Together, we will deduce the equations of the simplest mathematical model for this system, and we will rigorously solve it. So we'll do proofs.

But it turns out that, in the string, we will get a superposition of waves. We'll turn to a computer to simulate the solution we obtained and to "really" see the resonance phenomena between the oscillators and the string. We'll have to develop the computer algorithm ourselves because there is none available that I know of.

Then we will start asking questions like what happens if we add back into the mathematical model the phenomena that we neglected when we went for the "simplest model". Hence we will touch problems of waves propagating in inhomogeneous and nonlinear media very similar with the ones coming from Optics, Quantum Mechanics and Statistical Physics.

Weeks 7 and 8: Marta Lewicka

Introduction to Optimal Transportation

The objective of this course is to give a compact and rigorous introduction to the theory of optimal transportation, which is an important and lively field lying at the crossroads of probability theory, functional analysis, differential equations, and geometry.

No preparation beyond, say, honors analysis is necessary, and many examples and exercises will be provided. The tentative outline is as follows:

1) Formulation of the optimal transportation problem and first observations.
2) The Kantorovich problem and the Monge problem.
3) The discrete case.
4) Brenier's theorems.
5) The principal-agent problem and the Rochet-Chonet problem.