Let $f_1, \dots, f_l \in k[x_1, \dots, x_n, y_1, \dots, y_l]$, $f := (f_1, \dots, f_l)$, $X := f^{-1}(0) \subset \mathbb{A}^{n+l}$ (i.e., $X$ is the scheme of common zeros of $f_1, \dots, f_l$). Let $\Delta \subset X$ be the subscheme defined by the equation $\det \frac{\partial f}{\partial y} = 0$. Fix ~~...~~ a positive integer $N$.

Now let $Z$ be the scheme representing the following functor on the category of all $k$-algebras: an $R$-point of $Z$ is a pair consisting of a monic $q \in R[t]$ of degree $N$ and an element of $\varprojlim_r X(R[t]/(q^r))$ such that the scheme-theoretic preimage of $\Delta$ in $\operatorname{Spec} R[t]/(q^2)$ equals $\operatorname{Spec} R[t]/(q)$.

Proposition. $Z$ is a product of a $k$-scheme of finite type and an (infinite-dimensional) affine space.

This proposition is nice and simple, but it is not related directly to formal arcs (e.g., it doesn't involve the ring $R[[t]]$). When you pass to formal arcs, things become complicated.

Proof. By definition, $Z = \varprojlim_r Z_r$, where $Z_r$ $(r \geq 3)$ represents the following functor: an $R$-point of $Z_r$ is a triple $(q, \bar{x}, \bar{y})$, where

$q \in R[t]$ is monic of degree $N$,

$\bar{x} \in (R[t]/(q^r))^n$, $\bar{y} \in (R[t]/(q^{r-1}))^l$,

$f(\bar{x}, \bar{y}) \equiv 0 \mod q^{r-1}$,

$\det B \equiv 0 \mod q$, where $B := \frac{\partial f}{\partial y}(\bar{x}, \bar{y})$

$q^{-1} \cdot \det B$ is invertible modulo $q$,

$\hat{B} \cdot f(\bar{x}, \bar{y}) \equiv 0 \mod q^r$, where $\hat{B}$ is the adjugate matrix.

(The last condition makes sense despite the fact that $\bar{y}$ is defined only modulo $q^{r-1}$. To check this, use that $\hat{B}B = \det B$ is divisible by $q$.) Now it remains to prove the following lemma.

<u>Lemma</u> $Z_{r+1}$ is isomorphic to $Z_r \times \{\text{affine space}\}$ (isomorphism of schemes over $Z_r$).

The lemma is easily proved using "Newton's method" (i.e., calculus). ■

<u>The goal of my talks</u> is to explain that $Z_r$ is, in fact, the space of "nondegenerate" maps $A^1 \to X/\Gamma_r$, where $\Gamma_r$ is a certain smooth groupoid acting on $X$ and $X/\Gamma_r$ is the quotient <u>stack</u>. (You can guess the definition of $\Gamma_r$ if you wish.)

A broader goal is to explain that smooth groupoids acting on ~~sch~~ (possibly singular) schemes are nice and manageable objects.