Math 199 Introdution to Analysis and Linear Algebra, Section 43

Homework # 1 , due January 10

Problem 1

Get a copy of the notes: *"Foundations of Mathematical Analysis"* by Paul J. Sally. Read Chapter 1.1–1.4. and 1.6-1.7.

Problem 2

Let A, B, C be sets.

(1) Show that $\emptyset \subset A$.
(2) Show that $A \subset A$.
(3) Show that $A = B$ if and only if ($A \subset B$ and $B \subset A$).
(4) If $A \subset B$ and $B \subset C$ show that $A \subset C$.
(5) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
(6) $A \Delta B = \emptyset$ if and only if $A = B$.

Problem 3

Let A, B be subsets of some universal set X. Show the second DeMorgan's Law:

$$^c(A \cap B) = {}^cA \cup {}^cB.$$

Problem 4

Let X be a set and $R \subset X \times X$ a relation on X, which satisfies

(1) For all $a \in X$, $(a, a) \in R$, and
(2) for $a, b, c, \in X$, if $(a, b) \in R$ and $(b, c) \in R$, then $(c, a) \in R$.

Show that R is an equivalence relation.

Problem 5

Let $\mathbb{Z}$ be the integers. Suppose $a, b \in \mathbb{Z}$. We say that a divides b, denoted as $a|b$, if there exists an element $x \in \mathbb{Z}$ such that $b = ax$. Let $a, b, c \in \mathbb{Z}$. Prove the following

(1) If $a|b$ and $b|c$, then $a|c$.
(2) If $c|a$ and $c|b$ and $s, t \in \mathbb{Z}$, then $c|(sa + tb)$.

Problem 6

Let $n \in \mathbb{Z}$ be greater or equal to 2. Let $a, b \in \mathbb{Z}$. We say that *a is congruent to b modulo n*, denoted by $a \equiv b \mod n$, if $n|(b - a)$.

(1) Show that "congruence modulo n" is an equivalence relation on $\mathbb{Z}$.
(2) The equivalence classes of this equivalence relation are called *congruence classes modulo n*. Determine the number of congruence classes modulo n, find a set of representatives, and list the members of the congruence classes modulo 2.
(3) Let $a, b, c, d \in \mathbb{Z}$. Show that if $a \equiv b \mod n$ and $c \equiv d \mod n$, then $(a+b) \equiv (c+d) \mod n$ and $(ab) \equiv (cd) \mod n$.