ABSTRACTS

Geometry, Rigidity, and Group Actions

A conference in honor of Robert J. Zimmer's 60th Birthday

University of Chicago September 7–9, 2007

Shmuel Weinberger (Friday 9:45–10:30am)

The existence that topological rigidity is the uniqueness part of

Abstract. I will try to explain the why topological rigidity is closely related to existence of certain manifolds and orbifolds — and show some of the implications of this theory, and in particular (based on joint work with J. Block) give a counterexample to the Nielson realization problem and to a variant of a conjecture of Wall.

Shahar Mozes (Friday 11:15am–noon)

Invariant measures and stiffness for non-abelian groups of toral automorphisms

Abstract. In a joint work with Jean Bourgain, Alex Furman and Elon Lindenstrauss, we show that for a non elementary subgroup $\Gamma < \mathrm{SL}_2(\mathbb{Z})$ any Γ-invariant probability measure on the torus $\mathbb{T}^2$ is a convex combination of the normalized Haar measure and atomic probability measures supported on rational points. The same result holds for ν-stationary measures on $\mathbb{T}^2$ where ν is a probability measure supported on a finite generating set of Γ.

David Fisher (Friday 2:30–3:15pm)

Groups acting on manifolds

Abstract. In the 80's and 90's, Zimmer made a series of conjectures concerning group actions on manifolds, mostly concerned with actions of higher rank lattices and Lie groups. I will survey these conjectures, the original motivation for the conjectures in Zimmer's work, and recent progress. I will also state some more general conjectures and questions that now seem reasonable.

Leonid Polterovich (Friday 4:00–4:45pm)

Geometric structures on diffeomorphism groups

Abstract. I review geometric structures on various groups of diffeomorphisms (symplectic / contact / smooth) and discuss their applications to topology, dynamics and group actions.

Gregory A. Margulis (Saturday 9:30–10:15am)

Homogeneous flows and geometry of numbers

Abstract. The talk is based on the joint work with J. Athreya on the logarithm laws for unipotent flows. One of the aspects of this work is the following interesting phenomenon from the geometry of numbers. If B is a set of "large" measure in $\mathbb{R}^n$, $n > 2$, then the set of unimodular lattices which do not intersect B has "small" measure in the space of lattices.

Alex Furman (Saturday 11:00–11:45am)

Orbit Equivalence since Zimmer's Cocycle Superrigidity Theorem

Abstract. In this survey talk we shall discuss the impact of Zimmer's work in the theory of Orbit Equivalence in Ergodic Theory, and the recent developments in the area with the emphasis on rigidity phenomena and invariants of OE. The motivating question is "what can one learn about a group looking at the measurable orbit structure of a free ergodic probability measure preserving action of this group".

Sorin Popa (Saturday 2:30–3:15pm)

On the superrigidity of Bernoulli actions

Abstract. We present a general *cocycle superrigidity* result for Bernoulli actions of countable groups. We then use this result and a well known strategy of Zimmer to derive *orbit equivalence superrigidity* of Bernoulli actions of property (T) and product groups. We'll explain how these sharp rigidity phenomena are largely due to a deformation property of Bernoulli actions, called *malleability*.

Yehuda Shalom (Saturday 4:00–4:45pm)

Almost normal subgroups of arithmetic groups, and the structure of totally disconnected groups – On a question of Zimmer

Abstract. By a famous result of Margulis every normal subgroup of a 'higher rank' arithmetic group (such as $\mathrm{SL}(3,\mathbb{Z})$) is either finite or has finite index. In the late 70s Zimmer asked whether the conclusion remains true if the subgroup is only assumed 'almost normal' — its conjugates intersect it with finite index. For example, $\mathrm{SL}(3,\mathbb{Q})$ is a simple group, yet $\mathrm{SL}(3,\mathbb{Z})$ is almost normal in it.

We shall discuss a new conceptual approach to the problem, which remained quite mysterious so far. It turns out that for many groups (including $\mathrm{SL}(3,\mathbb{Z})$), a new fixed point property accounts for a positive answer to Zimmer's question, in analogy to geometric rigidity results for group actions on non-positively curved manifolds. A central ingredient is a surprisingly involved structure theory for totally disconnected groups (somewhat analogous to Lie theory for connected ones), developed by George Willis over the last decade. The relevant notions will be explained in the talk, which is based on joint work with George Willis.

Amos Nevo (Sunday 9:30–10:15am)

The ergodic theory of semisimple groups and lattice subgroups, and applications

Abstract. We will survey the recently established ergodic theorems for measure-preserving actions of semisimple algebraic groups and of their lattice subgroups. As an application we will then demonstrate how ergodic theorems lead to effective solutions to a variety of counting problems of lattice points in general subdomains of the group, and of some of its homogeneous spaces.

Alex Lubotzky (Sunday 11:00–11:45am)

Property 'tau' and hyperbolic manifolds

Abstract. The study of Property T w.r.t. a restricted class of representations (e.g. finite dimensional, finite, etc.). The latter (nowadays called Property 'tau') was initiated in a joint paper of Zimmer and the speaker ("Variants of Kazhdan's property for subgroups of semisimple groups" Israel J. Math. 1989).

While the motivation there was very different, this property has found various applications to geometry and especially to hyperbolic manifolds. We will survey some of these applications including a recent program led by Lackenby to use it for trying to prove the virtual Haken conjecture for hyperbolic 3-manifolds.

Post-conference workshop

Danijela Damjanovich (Sunday 1:30–2:00pm)
A quick review of two approaches to studying smooth perturbations of homogeneous abelian actions

Abstract. I will contrast two different methods applied to studying smooth perturbations of some standard classes of abelian homogeneous actions of higher rank (Weyl chamber flows, actions by automorphisms of nilmanifolds). One approach is classical (based on the iterative scheme due to Kolmogorov-Arnold-Moser) and the other is quite novel, based on robustness of certain invariant geometrical structures for the action. This is joint work with A. Katok.

Yves de Cornulier (Sunday 2:10–2:40pm)
Quasi-isometric embeddings of Lie groups into non-positively curved metric spaces

Abstract. We give a caracterization of connected Lie groups admitting a quasi-isometric embedding into a CAT(0) metric space.

Karin Melnick (Sunday 2:50–3:20pm)
An embedding theorem for automorphism groups of Cartan geometries

Abstract. I will present a theorem relating the automorphism group of a Cartan geometry to the group on which the geometry is modeled: a component of the adjoint representation of the first embeds in the adjoint representation of the second. The theorem and many of its consequences are analogous to, and improvements of, results of Zimmer, in particular the embedding theorem for automorphisms of G-structures.

In the study of geometric structures on manifolds, Cartan geometries provide a setting well-adapted to questions about homogeneity. Further results related to our embedding theorem are that sufficiently large rank or nilpotence degree of an automorphism group implies local homogeneity for many types of geometries.

These theorems are joint work with Uri Bader and Charles Frances.

Pierre Py (Sunday 3:50–4:20pm)
Bounded cohomology and dynamics on surfaces

Abstract. I will discuss the (non-)existence of area preserving actions of cocompact higher rank lattices on surfaces, following Zimmer's program. I would like to survey the possible applications to this program of some vanishing results by Burger and Monod in bounded cohomology, combined with some constructions of Gambaudo and Ghys. In particular, I will explain why the image of some higher rank lattice in the group of hamiltonian diffeomorphisms of a closed surface cannot contain the time 1 map of a hamiltonian flow.

Alireza Salehi Golsefidy (Sunday 4:30–5:00pm)
Lattices with small covolume

Abstract. In this talk we discuss lattices with "small" covolume in Chevalley groups over non-Archimedean fields. In the case of characteristic p, we show that up to isomorphism $\mathbf{G}\big(\mathbb{F}_p[t]\big)$ is the only lattice of minimum covolume in $\mathbf{G}\big(\mathbb{F}_p((1/t))\big)$. In the characteristic zero case, we study discrete transitive actions on the Bruhat-Tits building, and prove that there is no lattice in $\mathrm{PGL}(n, K)$ which acts transitively on the vertices of the Bruhat-Tits building if $n > 8$. We provide some partial results for smaller dimensions. Along the way a new proof of Siegel-Klingen theorem will be given.