2008 REU Problem Set 2: due July 2

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1. You just spilled some wine on the table in your favourite restaurant because someone told you the aggressive piglet joke. By applying subtle measurements with a chopstick, you realize that the diameter of the stain is 1. Show that you can cover it with your plate (having radius $1/\sqrt{3}$).

2. Show that for every c there are finitely many finite groups with c conjugacy classes.

3. Let G be a locally finite vertex transitive graph, let v be a vertex and let n be an even number. Show that the random walk of length n starting at v hits v with the highest probability.

4. Can you cover the space with disjoint circles of positive radius?

5. Show that you can paint the plane with finitely many colors such that no two points of distance 1 have the same color. How many colors do you need?

6. Let $X = \{0,1\}$ endowed with the uniform probability measure. Let

$$B = \{f : \mathbb{Z} \to X\}$$

be the set of 0-1 colorings of the integers, endowed with the product topology and product measure. Prove that if a measurable subset of B is invariant under the shift, then it has measure zero or its complement has measure zero.

7. The Invisible Flea does the kindness again, but now it starts jumping from an unknown point of $\mathbb{R}^n$ (still, with a constant jumping vector). For which values of n can you catch it for sure?

8. Is there a countably infinite, connected Haussdorff space?

9. A sequence of symbols is repetition free, if it does not contain the same segment twice, one right after the other. Is there an infinite repetition free sequence on 3 symbols?

10. Show that the additive group of $\mathbb{Q}$ does not have any minimal generating sets.