

2008 REU Problem Set 5: due July 17

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1. Is it true that a polynomial $f(x) \in \mathbb{Q}[x]$ considered as a function $f : \mathbb{Q} \rightarrow \mathbb{Q}$ is surjective (onto) if and only if it is linear?
2. You are given a triple of integers (a, b, c) . A step is to add an integer multiple of one of the entries to another entry. Show that you can always reach $(0, 0, d)$ in at most 100 steps.
3. Does the ABA decomposition holds with infinitely many rows and columns?
4. Is there an uncountable set of subsets of the natural numbers such that any two distinct sets have finite intersection?
5. Let $F = F_2$ be the free group on two generators. Show that the intersection of subgroups of finite index in F is trivial.
6. Are there 4 points on the plane such that all 6 possible distances are odd integers?
7. Prove that every large enough symmetric group can be generated by an element of order 2 and an element of order 3.
8. Is the (3-regular) honeycomb lattice a Cayley graph?
9. Is there a real function that takes on every real value in every open interval uncountably many times?
10. Show that if A is any set of points of size $d + 2$ in R^d then there exists a partition $A = B \cup C$ such that the convex hull of B intersects the convex hull of C .