

PS 10 Solutions
Math 256 Section 31

June 7, 2005

50.1) $2 (\mathbb{Q}(\sqrt{3}))$

50.2) $2 (\mathbb{Q}(i))$

50.9) $2 (\mathbb{Q}(\sqrt[3]{2}, i\sqrt{3})$ is the splitting field of $x^2 + 3$ over $\mathbb{Q}(\sqrt[3]{2})$)

50.23) a) $\text{irr}(i\sqrt{3}, \mathbb{Q}) = x^2 + 3$ and $\text{irr}(\sqrt[3]{2}, \mathbb{Q}(i\sqrt{3})) = x^3 - 2$, so $\{\mathbb{Q}(i\sqrt{3}) : \mathbb{Q}\} = 2$ and $\{\mathbb{Q}(\sqrt[3]{2}) : \mathbb{Q}(i\sqrt{3})\} = 3$, so $|G(E/\mathbb{Q})| = 2 \cdot 3 = 6$.

b) E is the splitting field of $x^3 - 2$ over \mathbb{Q} , so by exercise 22, $G(E/\mathbb{Q})$ is a subgroup of S_3 , but $|S_3| = 6$, so $G(E/\mathbb{Q}) = S_3$.

53.3) $|\lambda(\mathbb{Q})| = |G(K/\mathbb{Q})| = 8$

53.4) $|\lambda(\mathbb{Q}(\sqrt{2}, \sqrt{3}))| = [K : \mathbb{Q}(\sqrt{2}, \sqrt{3})] = 2$.

53.11) see answer page 510.