

PS 1 Solutions

Math 256 Section 31

April 16, 2005

I.E.7) Let λ be an eigenvalue for $\phi\psi$ with eigenvector $v \neq 0$.

Case 1 ($\lambda \neq 0$): $\phi\psi(v) = \lambda v$. So $\psi\phi\psi(v) = \psi(\lambda v) = \lambda\psi(v)$. $\psi(v) \neq 0$ (because $\lambda \neq 0$) so λ is an eigenvalue for $\psi\phi$ corresponding to $\psi(v)$.

Case 2a ($\lambda = 0$): If $\ker \phi \neq 0$ then let $u \neq 0 \in \ker \phi$, so $\psi\phi(u) = \psi(0) = 0 = \lambda u$. If not then ϕ is an isomorphism (this uses that V is finite-dimensional), so $\phi\psi(v) = 0 \iff \psi(v) = 0$. Let $u = \phi^{-1}(v)$. $v \neq 0$, so $u \neq 0$. Then $\psi\phi(u) = \psi(v) = 0 = \lambda v$.

II.E.2) Let $v \in K_i$. We want that $\phi(v) \in K_i$. From (1) we know that ϕ commutes with ψ_λ . $\psi_\lambda^i \phi = \psi_\lambda \psi_\lambda \dots \psi_\lambda \phi = \phi \psi_\lambda \psi_\lambda \dots \psi_\lambda = \phi \psi_\lambda^i$, so $\psi_\lambda^i \phi(v) = \phi \psi_\lambda^i(v) = \phi(0) = 0$. Thus $\phi(v) \in \ker \psi_\lambda^i$, as desired.

II.E.4) i) λ_1 : geometric = 1, algebraic = 2.

λ_2 : geometric = 1, algebraic = 1.

ii) 2: geometric = 2, algebraic = 2.

6: geometric = 1, algebraic = 1.

III.C.2) ii) The minimal polynomial is $x^4 - 1$.