PS 4 Solutions
Math 256 Section 31

April 27, 2005

III.E.2.ii) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ det = 9, trace = 7.

III.E.5) A matrix is diagonalizable iff its Jordan form is diagonal, but $J_n(r)$ is its own Jordan form and it is not diagonal if $n > 1$. ii and iv are diagonalizable (over $\mathbb{C}$).

III.E.8) Case 1: $\sigma(x) = (x - r_1)(x - r_2)(x - r_3)$, $r_1, r_2, r_3 \in \mathbb{C}$, all distinct. Then the Jordan form is $\begin{pmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{pmatrix}$.

Case 2: $\sigma(x) = (x - r_1)^2(x - r_2)$. If $m(x) = (x - r_1)(x - r_2)$ then the Jordan form is $\begin{pmatrix} r_1 & 0 & 0 \\ 0 & r_1 & 0 \\ 0 & 0 & r_2 \end{pmatrix}$ and if $m = \sigma$, then we have $\begin{pmatrix} r_1 & 1 & 0 \\ 0 & r_1 & 0 \\ 0 & 0 & r_2 \end{pmatrix}$

Case 3: $\sigma(x) = (x - r)^3$. If $m(x) = (x - r)$ then $\begin{pmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{pmatrix}$. If $m(x) = (x - r)^2$, then $\begin{pmatrix} r & 1 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{pmatrix}$. If $m = \sigma$, then $\begin{pmatrix} r & 1 & 0 \\ 0 & r & 1 \\ 0 & 0 & r \end{pmatrix}$