PS 8 Solutions
Math 256 Section 31

June 4, 2005

48.32) By Corollary 48.5, any isomorphism of $F(\alpha)$ into $\overline{F}$ that fixes F must send α to one of its conjugates. It also says that there is exactly one isomorphism for each conjugate, so the maximum number of such isomorphisms is the number of conjugates, which is $deg(\alpha, F) = n$.

49.1) The identity; $(\sqrt{2}, \sqrt{3}, \sqrt{5}) \mapsto (\sqrt{2}, -\sqrt{3}, -\sqrt{5})$.

49.2) $(\sqrt{2}, \sqrt{3}, \sqrt{5}) \mapsto (\sqrt{2}, -\sqrt{3}, \sqrt{5})$; $(\sqrt{2}, \sqrt{3}, \sqrt{5}) \mapsto (\sqrt{2}, \sqrt{3}, -\sqrt{5})$

49.4) The identity, ψ_{α_1,α_2}, and ψ_{α_1,α_3}.

49.9) σ is an isomorphism, so $\sigma^{-1} : \sigma[K] \to K$ is too. By Theorem 49.3, we can extend σ^{-1} to an isomorphism $\tau : K \to K$. Let $x \in K$, then σ^{-1} is onto so $\tau(x) \in K \Rightarrow \exists y \in \sigma[y]$ such that $\sigma^{-1}(y) = \tau(x)$, but τ is an extension of σ^{-1} and is one-to-one, so $x = y$, so $x \in \sigma[K]$, so $K \subset \sigma[K] \subset K$.

49.10) This problem only makes sense if $E \subset \overline{F}$. $\overline{F}$ is an algebraic extension of F and $F \subset E \subset \overline{F}$, so $\overline{F}$ is an algebraic extension of E, so $\overline{F}$ is an algebraic closure for E. Let τ be an isomorphism of E onto a subfield of $\overline{F}$ leaving F fixed. We can extend τ to $\sigma\overline{F} \to \overline{F}$, which by exercise 9, must be an automorphism.

49.11) E is an algebraic extension of F and $\overline{E}$ is an algebraic extension of E, so $\overline{E}$ is an algebraic extension of F, so $\overline{E}$ is an algebraic closure of F, so $\overline{F} \cong \overline{E}$.

49.13) Suppose K a field and $L = K(\alpha)$. From problem 48.32, we know that there are at most $\deg(\alpha, K)$ isomorphisms of L onto a subfield of $\overline{K}$ that fix K, i.e. $\{L : K\} \leq \deg(\alpha, K) = [L : K]$. E a finite extension of F so $E = F(\alpha_1, \ldots, \alpha_n)$. So $\{E : F\} = \{E : F(\alpha_1, \ldots, \alpha_{n-1})\}\{F(\alpha_1, \ldots, \alpha_{n-1}) : F(\alpha_1, \ldots, \alpha_{n-2})\} \ldots \{F(\alpha_1) : F\} \leq [E : F(\alpha_1, \ldots, \alpha_{n-1})][F(\alpha_1, \ldots, \alpha_{n-1}) : F(\alpha_1, \ldots, \alpha_{n-2})] \ldots [F(\alpha_1) : F] = [E : F]$.