Math 205
Additional Exercises

1. Prove theorem 9 for $\omega_2 = f_2 dx_2$ (see comment above formula (9.9).)

2. Let $I^2 = \{(r,\theta) \mid 0 \le r \le 1;\ 0 \le \theta \le 2\pi\}$. Let $\phi(r,\theta) = (r\cos\theta, r\sin\theta)$. Compute $\partial\phi$, and draw the picture. Show that in computing $\int_{\partial\phi}\omega$ for this particular ϕ, one can ignore certain pieces of $\partial\phi$.

3. Verify by direct calculation that $\int_{\partial\phi}\omega = \int_{\phi} d\omega$ for ϕ as in problem 2 above and the following ω:

$$dx; \quad x^n dx; \quad y dx; \quad x dy + y dx.$$

4. Let $I^2 = \{(u,v) \mid -\pi \le u \le +\pi;\ -1 \le v \le +1\}$. Define $\phi : I^2 \to \mathbb{R}^3$:

$$\phi(u,v) = \Big((2 + v\sin(u/2))\cos u, (2 + v\sin(u/2))\sin u, v\cos(u/2)\Big).$$

Draw a picture of $\phi(I^2)$. (If you get stuck, you can peek at fig. 13.13 in Wade.) Now draw $\varphi(I^2)$ where $\varphi(u,v) = (2\cos u, 2\sin u, v)$. Draw $\varphi(I^2)$. Compare $\phi(\partial I^2)$ and $\varphi(\partial I^2)$.

5. A 1-chain is a formal sum of paths, $\rho = \sum c_i \rho_i$ where $\rho_i : [a,b] \to \mathbb{R}^n$. (Note we fix a, b so they do not depend on i.) We say ρ is *closed* if again as formal sums

$$\sum c_i \rho_i(b) = \sum c_i \rho_i(a).$$

Let $\phi : I^2 \to \mathbb{R}^n$ be a 2-chain. Show how to parametrize $\phi(\partial I^2)$ so it is a 1-chain. Show this 1-chain is closed. Draw pictures to illustrate closed 1-chains and $\phi(\partial I^2)$.