

Part II. Calculus

1. Consider the function

$$y = (5 - x)^{1/3}$$

- What is the domain of the function?
- What is the image of the function?
- What is $f(x)$ at $x = 13$?
- What is $f'(x)$?
- What is $f'(x)$ at $x = 4$?
- Find the slope and the equation of the tangent line to the given curve at $(-3, 1)$.

2. Consider the function

$$f(x) = 4 - 3e^{2x} \quad (\text{for purposes of calculation use } e=3)$$

- Find $f'(x)$
- Find $f''(x)$
- Find $f'(3)$
- Find $f''(0)$

3. Let $f(x) = (5x^2 - 9x + 8) / (x^2 + 1)$. Calculate $f'(x)$ and simplify.

4. If $y = (x^3 + 5)^{4/3}$

- Find $\partial y / \partial x$.
- Find $\partial^2 y / \partial x^2$. Do not simplify.

5. Find dy/dx by implicit differentiation: $2x^3 + 5xy + 6y^2 = 87$

6. Use logarithmic differentiation to solve the following:

$$g(x) = (3x^5 - 4)(2x^3 + 9) / (7x^4 - 5)$$

6. Given the function $f(x) = 2x^3 + 3x^2 - 36x$

- Determine the intervals over which the function is increasing and decreasing.
- Determine the intervals over which the function is concave or convex.
- Determine all critical numbers and critical points.
- Identify which critical points are relative maxima and relative minima, and identify any inflection points.
- Using the above information, sketch the graph.

7. Evaluate the integral

$$\int_1^2 (4x^3 - x^2 - 2x + 3) \, dx$$

8. Find $\lim_{x \rightarrow 4} (3x^2 - 5x) / (x + 6)$

9. Find any vertical and horizontal asymptotes of the graph of the function
 $f(x) = (2x + 6) / (3x - 4)$

10. Suppose $f(x, y) = y^4 - 4xy + (1/2)x^2 - 20$

- a. What is $f'_x(x, y)$?
- b. What is $f'_y(x, y)$?
- c. What is f''_{xx} ?
- d. What is f''_{yy} ?
- e. What is f''_{xy} ?
- f. Find all critical numbers and points.
- g. Classify the critical points as relative maxima, relative minima, or inflection points.