

Algebra Placement Exam
Harris School of Public Policy
September 17, 2007

You have ninety minutes for this exam. No books, notes, calculators, or other aids are allowed. Please answer in the blue books provided, and please make sure to include your name and UCID number on all work submitted.

1. Linear Equations. (25 points, 5 each)

Consider the three points $P = (3, -2)$, $Q = (-4, 7)$, and $R = (-1, -1)$.

- (a) Find the equation of the line passing through points P and Q .
- (b) Find the y -intercept of the line passing through points P and Q .
- (c) Find the equation of the vertical line passing through point R .
- (d) Find the point of intersection of the lines in parts (a) and (c).
- (e) Give the equation of the line perpendicular to the line from part (a) which passes through the point R .

2. Quadratic Equations. (20 points, 5 each)

Consider the quadratic defined by the equation

$$y = -9 + 5x - x^2.$$

- (a) Find all intercepts of this parabola.
- (b) Find the vertex of this parabola.
- (c) Does this parabola open upward or downward? Explain.
- (d) Graph the parabola on a reasonable set of axes, being sure to label all points of interest.

3. Inequalities. (16 points, 8 each)

Find all solutions to the following inequalities. Express your answers in interval notation.

- (a) $4 - x \leq x + 6$
- (b) $7x + 2 < 3x^2$

4. Absolute Value. (16 points, 8 each)

- (a) Solve the inequality $|6 - 2x| > 5$, and express your answer in interval notation.
- (b) Graph the equation $y = |6 - 2x|$, making sure to indicate intercepts and other points of interest.

5. Solving Equations. (12 points, 6 each) Consider the polynomial defined by the equation

$$p(x) = x^3 + 8x^2 + 4x - 3.$$

- (a) Use the fact that $p(-1) = 0$ to factor this polynomial as completely as possible.
- (b) Find all real roots of this polynomial.

6. Functions and Graphing. (20 points, 10/5/5)

- (a) Find the domain, image, intercepts, and asymptotes of the function $f(x) = \frac{1}{\sqrt{x-4}}$, and graph f on a reasonable set of axes.
- (b) If $g(x) = x^2 + 2x + 1$, evaluate the expression $f(g(3))$.
- (c) What is the domain of the function $f \circ g$?

7. Simultaneous Equations. (16 points, 8 each)

Consider the following pair of simultaneous equations, where k is a constant:

$$\begin{array}{ll} \text{Equation 1:} & kx + 3y = 5 \\ \text{Equation 2:} & 4x - 2y = 1 \end{array}$$

- (a) Determine the value of k so that this pair of simultaneous equations has no common solution.
- (b) For the value $k = 1$, find the unique solution to these two simultaneous equations.

8. Exponents and Logarithms. (20 points, 5 each)

- (a) Use the rules of exponents to simplify the following expression:

$$\sqrt[3]{x} \cdot \frac{(x^{-3})^2}{x^{1/2} \cdot x^5}$$

- (Express your answer as a single power of x .)
- (b) Solve the following equation: $2^{x+2} \cdot 4^{3-x} = 8^x$
 - (c) Solve the following equation: $\log_{10} \sqrt{x} = -3$
 - (d) Graph the equation $y = \log_2(x - 1)$ on a reasonable set of axes.

9. Axioms for the Real Numbers. (10 points)

Use the axioms for the real numbers to give a formal step-by-step proof that the equation $-3x + 5 = -7$ has for its only solution the number $x = 4$.