

**Algebra Placement Exam**  
**Harris School of Public Policy**  
**September 5, 2008**

You have two hours for this exam. No books, notes, calculators, or other aids are allowed. Please answer in the blue books provided, and please make sure to include your name and UCID number on all work submitted.

**1. Lines.** (20 points, 5 each)

- (a) Find the equation of the line  $\ell_1$  with slope  $-\frac{2}{3}$  that passes through the point  $(-7, 4)$ .
- (b) Determine the  $y$ -intercept of the line  $\ell_1$ .
- (c) Find the equation of the line  $\ell_2$  that is perpendicular to  $\ell_1$  and that passes through the point  $(-7, 4)$ .
- (d) Find the equation of the vertical line  $\ell_3$  that passes through the point  $(-7, 4)$ .

**2. Parabolas.** (20 points, 5 each) Consider the parabola defined by the equation

$$y = -\frac{1}{8}x^2 + 3x + 6.$$

- (a) Find the  $x$ - and  $y$ -intercepts of this parabola.
- (b) Find the vertex of this parabola.
- (c) Write the equation for this parabola in Factored Form, that is, in the form  $y = a(x - x_1)(x - x_2)$ .
- (d) Graph the parabola on a reasonable set of axes, being sure to label all points of interest.

**3. Inequalities.** (20 points, 8/8/4)

Consider the inequality  $12 - x \leq x^2 + 7x - 8$ .

- (a) Find the solution set for the given inequality.
- (b) Graph the two equations  $y = 12 - x$  and  $y = x^2 + 7x - 8$  on the same set of axes.
- (c) Explain the relationship between your answers to parts (a) and (b).

**4. Absolute Value.** (16 points, 8 each)

- (a) Find all values of  $x$  that satisfy the equation  $|\frac{1}{2}x + 2| = x + 3$ .
- (b) Find all values of  $x$  that satisfy the inequality  $|\frac{1}{2}x + 2| < x + 3$ .

**5. Polynomials.** (16 points, 6/4/4) Consider the polynomial defined by  $p(x) = (x - 2)^5 + 28$ .

- (a) Write this polynomial in standard form (by multiplying it out or otherwise).
- (b) Compute  $p(0)$  and  $p(1)$ .
- (c) What can you conclude about the roots of  $p(x)$  based on your answer to (b)?

**6. Functions.** (16 points, 8 each)

- (a) Find the intercepts and asymptotes of the following function:  $f(x) = \frac{x + 1}{(x + 2)(x - 7)(2x + 3)}$
- (b) Find the domain and range/image of the following function:  $g(x) = \sqrt{\frac{4}{9}x^2 - 16}$

**7. Simultaneous Equations.** (16 points, 8 each) Consider the following pair of simultaneous equations:

$$\begin{array}{ll} \text{Equation 1:} & 2x + y = -7 \\ \text{Equation 2:} & 3x - y = 22 \end{array}$$

- (a) Find the solution to this pair of equations.
- (b) Graph these two lines and illustrate their point of intersection.

**8. Exponentials and Logarithms.** (20 points, 5 each)

- (a) Solve the following equation:  $32 = 4^{-2x+1}$
- (b) Solve the following equation:  $\log_3(x^3 - 1) = -2$
- (c) Simplify the following expression by writing it in the form  $a^k$  for some real number  $k$ :

$$\frac{a^{1/2} \cdot (a^2)^3}{a^{-5} \cdot \sqrt[3]{a^4}}$$

- (d) Graph the function  $y = -1 + 2^{1-x^2}$  on a reasonable set of axes.