

Algebra Placement Exam
Harris School of Public Policy
September 4, 2009

You have ninety minutes for this exam. No books, notes, calculators, or other aids are allowed. Please answer in the blue books provided, and please make sure to include your name and UCID number on all work submitted.

1. Linear Equations I. (20 points, 5 each)

Consider the points $A = (-6, 4)$, $B = (3, 1)$ and $C = (7, 13)$ in the Cartesian plane.

- (a) Find the equation of the line ℓ_1 that passes through A and B .
- (b) Find the equation of the line ℓ_2 that passes through B and C .
- (c) Are the lines ℓ_1 and ℓ_2 perpendicular? Explain.
- (d) Find the area of the triangle $\triangle ABC$.

2. Linear Equations II. (20 points, 5/10/5)

Consider the following two lines:

$$\begin{array}{ll} \text{Line 1:} & 2x + 3y = 4 \\ \text{Line 2:} & x + 4y = 21 \end{array}$$

- (a) Find the vertical distance between these two lines along the line $x = -4$.
- (b) Find the point of intersection P of these two lines.
- (c) Find the equation of the line through the point P with slope $m = -\frac{1}{2}$.

3. Quadratics. (20 points, 10 each) Consider the parabola defined by the equation

$$y = \frac{1}{2}x^2 - x - 4.$$

- (a) Solve the inequality $\frac{1}{2}x^2 - x - 4 \leq 8$.
- (b) Graph the parabola $y = \frac{1}{2}x^2 - x - 4$, being sure to indicate the vertex and any intercepts.

4. Absolute Values and Inequalities. (24 points, 8 each)

Find the solution sets for each of the following equation and inequalities:

- (a) $|2x + 4| = 6 + |x|$
- (b) $\frac{25}{5 - x} \leq 10$
- (c) $|3x - 8| > \frac{1}{10}$

5. Polynomials. (16 points, 4 each)

Consider the polynomial defined by $p(x) = (x + 2)^4 - (x - 2)^4$.

- (a) Write this polynomial in standard form.
- (b) What is the degree of this polynomial?
- (c) Find the complete factorization of this polynomial (with real coefficients).
- (d) Find all real roots of $p(x)$, and explain how you know you have found them all.

6. Graphing. (20 points, 15/5)

Consider the following three inequalities:

Inequality 1:	$y \leq 16 - x^2$
Inequality 2:	$y \geq 2x + 1$
Inequality 3:	$ x > 2$

- (a) Graph the set of points in the Cartesian plane that satisfy all three inequalities.
- (b) Find the minimum y -value of any point that satisfies the three inequalities.

7. Exponentials and Logarithms. (21 points, 7 each)

- (a) Put the following four real numbers in increasing order: 120 , $(\frac{1}{3})^{-5}$, $16^{3/2}$, $\sqrt[3]{1,000,000}$
- (b) Find the value of x that satisfies the following equation:

$$\log_5 \left(\frac{1}{625} \right) + \log_3 x = \log_2 32$$

- (c) Find the value(s) of k that satisfy the following equation for all values of $a \neq 0$:

$$\frac{a^{-7/4} \cdot (a^{1/4})^{-6} \cdot a^{k^2}}{a^5 \cdot \sqrt[4]{a^3}} = 1$$

8. Algebra. (16 points, 4 each)

Which of the following expressions are algebraically equivalent to $\frac{1}{a+1}$?

(For each, answer "yes" or "no.")

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|---|-------------------------|
| (a) $\frac{1}{\sqrt{a^2+1}}$ | (c) $\frac{a-1}{a^2-1}$ |
| (b) $1 - a + a^2 - a^3 + a^4 - a^5 + \dots$ | (d) $(a+1)^{-1}$ |