Rules of Arithmetic for the Real Numbers $\mathbb{R}$

E1. (Reflexivity, Symmetry, and Transitivity of Equality)

Reflexivity of Equality	If $a \in \mathbb{R}$, then $a = a$.
Symmetry of Equality	If $a, b \in \mathbb{R}$ and $a = b$, then $b = a$.
Transitivity of Equality	If $a, b, c \in \mathbb{R}$ and $a = b$ and $b = c$, then $a = c$.

E2. (Additive Property of Equality)

If $a, b, c \in \mathbb{R}$ and $a = b$, then $a + c = b + c$.

E3. (Multiplicative Property of Equality)

If $a, b, c \in \mathbb{R}$ and $a = b$, then $a \cdot c = b \cdot c$.

A1. (Closure of Addition)

If $a, b \in \mathbb{R}$, then $a + b \in \mathbb{R}$.

A2. (Associativity of Addition)

If $a, b, c \in \mathbb{R}$, then $(a + b) + c = a + (b + c)$.

A3. (Commutativity of Addition)

If $a, b \in \mathbb{R}$, then $a + b = b + a$.

A4. (Additive Identity)

There is an element $0 \in \mathbb{R}$ such that $a + 0 = a$ and $0 + a = a$ for every $a \in \mathbb{R}$.

A5. (Additive Inverses)

For each element $a \in \mathbb{R}$, there is an element $-a \in \mathbb{R}$ such that $a + (-a) = 0$ and $(-a) + a = 0$.

M1. (Closure of Multiplication)

If $a, b \in \mathbb{R}$, then $a \cdot b \in \mathbb{R}$.

M2. (Associativity of Multiplication)

If $a, b, c \in \mathbb{R}$, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

M3. (Commutativity of Multiplication)

If $a, b \in \mathbb{R}$, then $a \cdot b = b \cdot a$.

M4. (Multiplicative Identity)

There is an element $1 \in \mathbb{R}$ (with $1 \neq 0$) such that $a \cdot 1 = a$ and $1 \cdot a = a$ for every $a \in \mathbb{R}$.

M5. (Multiplicative Inverses)

For each element $a \in \mathbb{R}$ with $a \neq 0$, there is an element $a^{-1} \in \mathbb{R}$ such that $a \cdot a^{-1} = 1$ and $a^{-1} \cdot a = 1$.

D. (Distributivity of Multiplication over Addition)

If $a, b, c \in \mathbb{R}$, then $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(a + b) \cdot c = a \cdot c + b \cdot c$.

O1. (Transitivity of Inequality)

If $a, b, c \in \mathbb{R}$ and $a < b$ and $b < c$, then $a < c$.

O2. (Trichotomy)

If $a, b \in \mathbb{R}$, then exactly one of the following is true: $a < b$, $a = b$, or $a > b$.

O3. (Additive Property of Inequality)

If $a, b, c \in \mathbb{R}$ and $a < b$, then $a + c < b + c$.

O4. (Multiplicative Property of Inequality)

If $a, b, c \in \mathbb{R}$ and $a < b$ and $c > 0$, then $a \cdot c < b \cdot c$.

LUB. (Least Upper Bound Property)

If S is a non-empty set of real numbers (that is, $S \neq \emptyset$) that is bounded above (that is, there is some $a \in \mathbb{R}$ such that if $x \in S$, then $x \leq a$), then the set S has a least upper bound (that is, there is some $b \in \mathbb{R}$ that is an upper bound for S, and if a is any other upper bound for S, then $b \leq a$).