

**Calculus Placement Exam**  
**Harris School of Public Policy**  
**September 17, 2007**

You have ninety minutes for this exam. No books, notes, calculators, or other aids are allowed. Please answer in the blue books provided, and please make sure to include your name and UCID number on all work submitted.

**1. Limits.** (12 points, 6 each)

(a) Evaluate the limit:  $\lim_{x \rightarrow -2} (x^2 + 5x - 1)$ .

(b) Evaluate the limit:  $\lim_{x \rightarrow 4} \frac{x^2 - 7x + 12}{x - 4}$ .

**2. Continuity.** (12 points, 6 each)

(a) Define what it means for a function  $f$  to be *continuous at the point*  $x = a$ .

(b) Is the function  $f(x) = |x - 3|$  continuous at the point  $x = 3$ ? Explain.

**3. Differentiability.** (16 points, 8 each)

(a) Define what it means for the function  $f$  to be *differentiable at the point*  $x = a$ .

(b) Use the definition of the derivative to compute  $f'(a)$  for the function  $f(x) = x^3 - x$ .

**4. Derivatives.** (24 points, 8 each)

For each of the following, find  $\frac{dy}{dx}$ :

(a)  $y = (x^2 + 3x)^{5/2}$

(b)  $y = \frac{x}{1 + \ln x}$

(c)  $y^2 + xy + x^2 = e^x$

**5. Tangent Lines.** (12 points)

Find the equation of the line tangent to the curve  $y = \frac{1}{2}x^3 - \frac{1}{x}$  when  $x = 2$ .

**6. Optimization.** (12 points)

Suppose the sum of three positive numbers is 24 and that the first number is three times the second one. Find (with explanation) the maximum product of these three numbers.

**7. Analysis of Functions.** (30 points, 5 each)

Consider the function  $f(x) = x^2 \cdot e^{-x}$ .

- (a) What is the domain of  $f$ ?
- (b) Identify all critical points of  $f$ ,
- (c) Find the intervals on which  $f$  is increasing and decreasing.
- (d) Determine all local maxima and minima of  $f$ .
- (e) Find the intervals on which  $f$  is concave up and concave down.
- (f) Determine all inflection points of  $f$ .

**8. Asymptotes.** (15 points, 5 each)

Consider the function  $f(x) = \frac{3-x}{2x+4}$ .

- (a) Identify any horizontal asymptotes of  $f$ .
- (b) Identify any vertical asymptotes of  $f$ .
- (c) For each vertical asymptote, evaluate the left- and right-hand limits of  $f$  at the asymptote.