You have ninety minutes for this exam. No books, notes, calculators, or other aids are allowed. Please answer in the blue books provided, and please make sure to include your name and UCID number on all work submitted.

1. Limits. (21 points, 7 each)
   (a) For the function \( f(x) = -4x + 2 \), what value(s) of \( \delta > 0 \) will guarantee that if \( 0 < |x - 3| < \delta \), then \( |f(x) + 10| < \frac{1}{10} \)?
   (b) Evaluate the limit: \( \lim_{x \to 1^+} \frac{x^3}{x^2 - x} \).
   (c) Evaluate the limit: \( \lim_{x \to -3} \frac{x + 3}{x + 2} \).

2. Continuity. (14 points, 7 each)
   (a) Give an example of a function that is defined in a neighborhood of the point \( x = 4 \) (including at the point \( x = 4 \) itself) but is not continuous at \( x = 4 \).
   (b) Is the function \( f(x) = |3x + 17| \) continuous at \( x = -\frac{17}{3} \)? Explain.

3. Differentiability. (16 points, 8 each)
   (a) Define what it means for a function \( f(x) \) to be differentiable at the point \( x = a \).
   (b) Use the definition of the derivative to compute \( f'(a) \) for the function \( f(x) = \frac{1}{x^3} \).

4. Derivatives. (24 points, 8 each)
   For each of the following, find \( \frac{dy}{dx} \):
   (a) \( y = 2x \cdot e^{-x} \)
   (b) \( y = \frac{1}{\ln x} \)
   (c) \( xy^2 - y^3 = 1 \)

5. Tangent Lines. (12 points)
   Find the equation of the line tangent to the curve \( y = \frac{x}{1 + x^2} \) at the point \( x = 3 \).
6. **Optimization.** (15 points)

Find the maximum and minimum values of \( f(x) = (2x^2 - 3x + 2)^{1/3} \) on the interval \([0, 1]\).

7. **Analysis of Functions I.** (35 points, 5 each)

Consider the function \( f(x) = x^2 \cdot \ln(1 + x^2) \).

(a) Identify all critical points of \( f \).
(b) Find the intervals on which \( f \) is increasing and decreasing.
(c) Determine all local maxima and minima of \( f \).
(d) Find the intervals on which \( f \) is concave up and concave down.
(e) Determine all inflection points of \( f \).
(f) Draw an accurate graph of \( y = f(x) \).

8. **Analysis of Functions II.** (35 points, 5 each)

Consider the function \( f(x) = x^2 \cdot \ln(1 + x^2) \).

(a) Identify all critical points of \( f \).
(b) Find the intervals on which \( f \) is increasing and decreasing.
(c) Determine all local maxima and minima of \( f \).
(d) Find the intervals on which \( f \) is concave up and concave down.
(e) Determine all inflection points of \( f \).
(f) Draw an accurate graph of \( y = f(x) \).

9. **Asymptotes.** (15 points, 5 each)

Consider the following function: \( f(x) = \frac{4x^3 - 4}{(x - 1)(x + 3)^2} \)

(a) Identify any horizontal asymptotes of \( f \).
(b) Identify any vertical asymptotes of \( f \).
(c) For each vertical asymptote, evaluate the left- and right-hand limits of \( f \) at the asymptote.

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