# Definitions and Theorems 1

Some Elementary Theorems from Algebra of the Real Numbers

**Theorem.** If $a, b \in \mathbb{R}$, then $(a - b)(a + b) = a^2 - b^2$.

**Theorem.** If $a, b \in \mathbb{R}$, then $(a + b)(a + b) = a^2 + 2ab + b^2$.

**Theorem.** If $a, b \in \mathbb{R}$ and $a \cdot b = 0$, then $a = 0$ or $b = 0$.

**Theorem.** If $a, b \in \mathbb{R}$ and $a \neq 0$, then the equation $ax + b = 0$ has solution $x = -\dfrac{b}{a}$.

**Theorem.** (Quadratic Formula)

If $a, b, c \in \mathbb{R}$ and $a \neq 0$, then the equation $ax^2 + bx + c = 0$ has solutions $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Definitions and Theorems concerning Absolute Value

**Definition.** If $x \in \mathbb{R}$, then $|x| = \begin{cases} x & , \text{ if } x \geq 0 \\ -x & , \text{ if } x \leq 0. \end{cases}$

**Theorem.** If $a, b \in \mathbb{R}$, then $|a| < b$ if and only if $-b < a < b$.

**Theorem.** If $a, b \in \mathbb{R}$, then $|a| > b$ if and only if either $a > b$ or $-a > b$.

Definitions of Intervals on the Real Line

$(a, b) = \{x \in \mathbb{R} \mid a < x < b\}$

$[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$

$(a, b] = \{x \in \mathbb{R} \mid a < x \leq b\}$

$[a, b) = \{x \in \mathbb{R} \mid a \leq x < b\}$

$(a, +\infty) = \{x \in \mathbb{R} \mid a < x\}$

$[a, +\infty) = \{x \in \mathbb{R} \mid a \leq x\}$

$(-\infty, b) = \{x \in \mathbb{R} \mid x < b\}$

$(-\infty, b] = \{x \in \mathbb{R} \mid x \leq b\}$