Definitions and Theorems 10

Analysis of Functions: Increasing/Decreasing, Maxima/Minima, Critical Points
The Mean-Value Theorem and Its Consequences

Definition. (Increasing/Decreasing)

A function f is said to be *increasing* on the interval I provided that, given any $x, y \in I$ with $x < y$, we have $f(x) < f(y)$.
A function f is said to be *decreasing* on the interval I provided that, given any $x, y \in I$ with $x < y$, we have $f(x) > f(y)$.

Definition. (Local Maximum/Minimum) Let $c \in \mathrm{Dom}(f)$.

The point c is said to be a *local maximum* of f if there is some interval I such that $c \in I$ and, if $x \in I \cap \mathrm{Dom}(f)$, then $f(x) \leq f(c)$.
The point c is said to be a *local minimum* of f if there is some interval I such that $c \in I$ and, if $x \in I \cap \mathrm{Dom}(f)$, then $f(x) \geq f(c)$.

Theorem.

Suppose f is continuous on the interval $[a, b]$.
Then there is some $c \in [a, b]$ such that $f(x) \leq f(c)$ for every $x \in [a, b]$.

Theorem. (Rolle's Theorem)

Let f be continuous on $[a, b]$ and differentiable on (a, b).
Suppose $f(a) = f(b)$. Then there is some $c \in (a, b)$ such that $f'(c) = 0$.

Theorem. (Mean Value Theorem)

Let f be continuous on $[a, b]$ and differentiable on (a, b).
Then there is some $c \in (a, b)$ such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

Theorem.

If $f'(c) > 0$ for every $c \in (a, b)$, then f is increasing on (a, b).
If $f'(c) < 0$ for every $c \in (a, b)$, then f is decreasing on (a, b).

Theorem/Definition.

Let f be defined on an interval I, and suppose c is a local maximum or local minimum of f on I. Then c belongs to one of the following three types of points:

I.	*Stationary Points:*	$f'(c) = 0$
II.	*Singular Points:*	$f'(c)$ is undefined.
III.	*Endpoints:*	If $I = [a, b]$, then $c = a$ or $c = b$.

A point which falls into any of these three categories is called a *critical point.*