Definitions and Theorems 6

Differentiation

Theorem. (Chain Rule) Suppose f is defined on the interval (a, b) and is differentiable at $c \in (a, b)$. Suppose g defined on an interval containing $f(c)$ and is differentiable at $f(c)$. Then the composite function $g \circ f$ defined by $(g \circ f)(x) = g(f(x))$ is differentiable at $x = c$ and

$$(g \circ f)'(c) = g'(f(c)) \cdot f'(c).$$

The Number e and the Natural Logarithm

Definition. (The number e) The number e is the unique positive real number such that $\lim_{h \to 0} \dfrac{e^h - 1}{h} = 0$.

Theorem. (Facts about e)

(a) $e \approx 2.71828$

(b) $\lim_{n \to \infty} \left(1 + \dfrac{1}{n}\right)^n = e$.

(c) $e = 1 + \dfrac{1}{1!} + \dfrac{1}{2!} + \dfrac{1}{3!} + \dfrac{1}{4!} + \cdots = \sum_{n=0}^{\infty} \dfrac{1}{n!}$

Definition. (The Natural Logarithm) The **natural logarithm** is the function defined by

$$\ln x = \log_e x.$$

That is, $y = \ln x$ if and only if $e^y = x$.

Theorem. (Differentiation of Exponentials) If $f(x) = e^x$, then $f'(x) = e^x$.

Theorem. (Differentiation of Logarithms) If $f(x) = \ln x$, then $f'(x) = \frac{1}{x}$.