

Elementary Number Theory  
Math 175, Section 30  
Autumn Quarter 2008  
Written Exercises from Week 7

**Exercise 0.0.1** Prove that if  $n$  is an integer, then the number  $\frac{1}{5}n^5 + \frac{1}{3}n^3 + \frac{7}{15}n$  is also an integer.

**Exercise 0.0.2** The Eggs in Baskets Problem

“A farmer is taking her eggs to the market in a cart, but she hits a pothole, which knocks over all the containers of eggs. Though she is unhurt, every egg is broken. So she goes to her insurance agent, who asks her how many eggs she had. She says she doesn’t know, but she remembers something from various ways she tried packing the eggs. When she put the eggs in groups of two, three, four, five, and six there was one egg left over, but when she put them in groups of seven they ended up in complete groups with no eggs left over.”

1. What is the minimum number of eggs that the farmer had?
2. Find the complete set of possible solutions, expressed as a congruence.

**Exercise 0.0.3** Archimedes’ Cattle Problem

If thou art diligent and wise, O stranger, compute the number of cattle of the Sun, who once upon a time grazed on the fields of the Thrinacian isle of Sicily, divided into four herds of different colours, one milk white, another a glossy black, a third yellow and the last dappled. In each herd were bulls, mighty in number according to these proportions: Understand, stranger, that the white bulls were equal to a half and a third of the black together with the whole of the yellow, while the black were equal to the fourth part of the dappled and a fifth, together with, once more, the whole of the yellow. Observe further that the remaining bulls, the dappled, were equal to a sixth part of the white and a seventh, together with all of the yellow. These were the proportions of the cows: The white were precisely equal to the third part and a fourth of the whole herd of the black; while the black were equal to the fourth part once more of the dappled and with it a fifth part, when all, including the bulls, went to pasture together. Now the dappled in four parts were equal in number to a fifth part and a sixth of the yellow herd. Finally the yellow were in number equal to a sixth part and a seventh of the white herd. If thou canst accurately tell, O stranger, the number of cattle of the Sun, giving separately the number of well-fed bulls and again the number of females according to each colour, thou wouldst not be called unskilled or ignorant of numbers, but not yet shalt thou be numbered among the wise.

But come, understand also all these conditions regarding the cattle of the Sun. When the white bulls mingled their number with the black, they stood firm, equal in depth and breadth, and the plains of Thrinacia, stretching far in all ways, were filled with their multitude. Again, when the yellow and the dappled bulls were gathered into one herd they stood in such a manner that their number, beginning from one, grew slowly greater till it completed a triangular figure, there being no bulls of other colours in their midst nor none of them lacking. If thou art able, O stranger, to find out all these things and gather them together in your mind, giving all the relations, thou shalt depart crowned with glory and knowing that thou hast been adjudged perfect in this species of wisdom.