

Analysis in \mathbb{R}^n
Math 203, Section 30
Autumn Quarter 2007
Written Exercises from Thursday, October 25

Exercise 0.0.1 Find a basis for the vector space \mathbb{C}^n over \mathbb{R} , and determine its dimension.

Exercise 0.0.2 Prove the Cauchy-Schwarz Inequality for \mathbb{C}^n .

(Hint: Expand the expression $\langle z - \lambda w, z - \lambda w \rangle$ and choose λ judiciously.)

Exercise 0.0.3 Prove the Hölder Inequality for \mathbb{C}^n .

Exercise 0.0.4 Prove the Triangle Inequality for $\ell_n^p(\mathbb{C})$.

Exercise 0.0.5 Why is \mathbb{Q}^n not an interesting vector space in which to do analysis?

- i.* Show that the Intermediate Value Theorem does not hold for continuous functions $f : \mathbb{Q} \rightarrow \mathbb{Q}$.
- ii.* Show that there exists a continuous function $f : [0, 1] \cap \mathbb{Q} \rightarrow \mathbb{Q}$ that does not attain a maximum.
- iii.* Show that, given a point $\mathbf{p} \in \mathbb{Q}^n$ and a hyperplane $H \subset \mathbb{Q}^n$, there is not necessarily a point on H that is nearest to \mathbf{p} .