Analysis in $\mathbb{R}^n$
Math 203, Section 30
Autumn Quarter 2007
Written Exercises from Thursday, October 25

Exercise 0.0.1 Find a basis for the vector space $\mathbb{C}^n$ over $\mathbb{R}$, and determine its dimension.

Exercise 0.0.2 Prove the Cauchy-Schwarz Inequality for $\mathbb{C}^n$.
(Hint: Expand the expression $\langle z - \lambda w, z - \lambda w \rangle$ and choose $\lambda$ judiciously.)

Exercise 0.0.3 Prove the H"older Inequality for $\mathbb{C}^n$.

Exercise 0.0.4 Prove the Triangle Inequality for $\ell^n_p(\mathbb{C})$.

Exercise 0.0.5 Why is $\mathbb{Q}^n$ not an interesting vector space in which to do analysis?
   i. Show that the Intermediate Value Theorem does not hold for continuous functions $f : \mathbb{Q} \to \mathbb{Q}$.
   ii. Show that there exists a continuous function $f : [0, 1] \cap \mathbb{Q} \to \mathbb{Q}$ that does not attain a maximum.
   iii. Show that, given a point $p \in \mathbb{Q}^n$ and a hyperplane $H \subset \mathbb{Q}^n$, there is not necessarily a point on $H$ that is nearest to $p$. 