Analysis in $\mathbb{R}^n$
Math 203, Section 30
Autumn Quarter 2007
Written Exercises from Thursday, November 1

Exercise 0.0.1 Let (X, d) be a metric space, and let $Y \subseteq X$. Show that (Y, d) is a metric space. (In this case, we say that Y has the *induced metric* from X.)

Exercise 0.0.2 Consider a sequence of points $\{x_n\}_{n=1}^{\infty}$ in $\mathbb{R}^n$. Show that $\{x_n\}$ converges in $\ell_n^p(\mathbb{R})$ if and only if it converges in $\ell_n^q(\mathbb{R})$, for any $p, q \in [1, +\infty]$.

Exercise 0.0.3 Let (X, d) be a metric space, and let $x, y \in X$. Show that there exist open balls, $B_{r_x}(x)$ and $B_{r_y}(y)$ such that $B_{r_x}(x) \cap B_{r_y}(y) = \emptyset$. (This condition tells us that X is *Hausdorff*.)

Exercise 0.0.4 Consider the set $\mathbb{R}^n$, and define $||x||_p = \left(\sum_{i=1}^n |x_i|^p\right)^{1/p}$.

i. Does $||\cdot||_p$ define a norm when $0 < p < 1$? Explain.

ii. Describe the set $B = \{x \in \mathbb{R}^n \mid ||x||_p < 1\}$ when $0 < p < 1$.

Exercise 0.0.5 Compute the volume of the unit ball in $\ell_n^1(\mathbb{R})$ and $\ell_n^{\infty}(\mathbb{R})$.