Exercise 0.0.1 Let \((X, d)\) be a metric space, and let \(Y \subseteq X\). Show that \((Y, d)\) is a metric space. (In this case, we say that \(Y\) has the induced metric from \(X\).)

Exercise 0.0.2 Consider a sequence of points \(\{x_n\}_{n=1}^{\infty}\) in \(\mathbb{R}^n\). Show that \(\{x_n\}\) converges in \(\ell_p^n(\mathbb{R})\) if and only if it converges in \(\ell_q^n(\mathbb{R})\), for any \(p, q \in [1, +\infty]\).

Exercise 0.0.3 Let \((X, d)\) be a metric space, and let \(x, y \in X\). Show that there exist open balls, \(B_{r_x}(x)\) and \(B_{r_y}(y)\) such that \(B_{r_x}(x) \cap B_{r_y}(y) = \emptyset\). (This condition tells us that \(X\) is Hausdorff.)

Exercise 0.0.4 Consider the set \(\mathbb{R}^n\), and define \(\|x\|_p = (\sum_{i=1}^{n} |x_i|^p)^{1/p}\).

i. Does \(\| \cdot \|_p\) define a norm when \(0 < p < 1\)? Explain.

ii. Describe the set \(B = \{x \in \mathbb{R}^n \mid \|x\|_p < 1\}\) when \(0 < p < 1\).

Exercise 0.0.5 Compute the volume of the unit ball in \(\ell_1^n(\mathbb{R})\) and \(\ell_\infty^n(\mathbb{R})\).