Analysis in $\mathbb{R}^n$
Math 203, Section 30
Autumn Quarter 2007
Written Exercises from Tuesday, November 13

Exercise 0.0.1 Recall that a *topology* $\mathcal{T}$ on a set X is a subset of $\mathcal{P}(X)$ that satisfies the following properties:

1. If $\{U_i\}_{i\in I} \subset \mathcal{T}$, then $\bigcup_{i\in I} U_i \in \mathcal{T}$
2. If $U_1, U_2, \ldots, U_n \in \mathcal{T}$, then $\bigcap_{i=1}^{n} U_i \in \mathcal{T}$
3. $\emptyset, X \in \mathcal{T}$

The sets in $\mathcal{T}$ are called the *open sets* of the topology.

i. Show that if X is any set, then $\mathcal{T} = \{\emptyset, X\}$ is a topology on X. (This topology is called the *indiscrete topology.*)

ii. If X is a set, is the indiscrete topology on X metrizable, that is, does there exist a metric on X such that the open sets of the topology are the same as the open sets in the metric space?

Exercise 0.0.2 One might be inclined to think that if X is a metric space and A is a proper subset of X (that is, $A \neq \emptyset$ and $A \neq X$), then the boundary of A should be non-empty. When one considers discrete metric spaces, one quickly realizes that this is not true (the boundary of *any* subset of a discrete metric space is empty), but this is not the only exception. Find an example of a non-discrete metric space X and a proper subset $A \subset X$ such that $\partial A = \emptyset$.