Analysis in $\mathbb{R}^n$
Math 203, Section 30
Autumn Quarter 2007
Written Exercises from Tuesday, November 20

Exercise 0.0.1 Prove the result in Exercise 0.3.8 using induction.

Exercise 0.0.2 Prove the result in Exercise 0.3.9 using induction.

Exercise 0.0.3 Let $\ell^\infty(\mathbb{R}) = \{(a_n)_{n\in\mathbb{N}} \mid \sup_{n\in\mathbb{N}}|a_n| < \infty\}$ be the set of bounded sequences of real numbers.

i. Check that $\ell^\infty(\mathbb{R})$ is a vector space over $\mathbb{R}$ with addition and scalar multiplication defined as follows:

$$(a_n) + (b_n) = (a_n + b_n)$$
$$c \cdot (a_n) = (c \cdot a_n).$$

(Don't actually turn this part in.)

ii. Show that $\ell^\infty(\mathbb{R})$ has a norm defined by

$$||(a_n)|| = \sup_{n\in\mathbb{N}}|a_n|$$

and hence is a metric space with the distance function $d((a_n),(b_n)) = ||(a_n) - (b_n)||$.

iii. Find a bounded sequence in $\ell^\infty(\mathbb{R})$ that does not have a convergent subsequence.
(Compare this with the results of 0.3.9 and 0.3.10.)