

Analysis in  $\mathbb{R}^n$   
Math 205, Section 30  
Spring Quarter 2008  
Written Exercises from Week 1

**Exercise 0.0.1** If  $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is of class  $C^k$  and  $g : \mathbb{R}^m \rightarrow \mathbb{R}^\ell$  is of class  $C^j$ , then what is the best result that can be stated about the composition  $g \circ f : \mathbb{R}^n \rightarrow \mathbb{R}^\ell$ ?

**Exercise 0.0.2** Let  $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ . Show that our definition of what it means for  $f$  to be  $C^1$  is equivalent to the idea that  $Df : \mathbb{R}^n \rightarrow \mathcal{L}(\mathbb{R}^n, \mathbb{R}^m)$  is continuous, where  $\mathcal{L}(\mathbb{R}^n, \mathbb{R}^m)$  is the collection of linear maps from  $\mathbb{R}^n$  to  $\mathbb{R}^m$ , which is given the usual norm.