Analysis in $\mathbb{R}^n$
Math 205, Section 30
Spring Quarter 2008
Written Exercises from Week 2

Exercise 0.0.1  Decide whether it is possible to solve the equations

$$xu^2 + yzv + x^2z = 3$$
$$xyv^3 + 2zu - v^2v^2 = 2$$

for $(u,v)$ near $(1,1)$ as a function of $(x,y,z)$ near $(1,1,1)$.

Exercise 0.0.2

i. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be a diffeomorphism with inverse $g : \mathbb{R}^2 \to \mathbb{R}^2$, and let $J = Df$. Show that:

$$\frac{\partial g_1}{\partial y_1} = \frac{1}{J} \frac{\partial f_2}{\partial x_2}$$
$$\frac{\partial g_1}{\partial y_2} = -\frac{1}{J} \frac{\partial f_1}{\partial x_2}$$
$$\frac{\partial g_2}{\partial y_1} = -\frac{1}{J} \frac{\partial f_2}{\partial x_1}$$
$$\frac{\partial g_2}{\partial y_2} = \frac{1}{J} \frac{\partial f_1}{\partial x_1}$$

ii. Do this computation explicitly for $f(x_1, x_2) = (x_1^2 - x_2^2, 2x_1x_2)$. 