

Analysis in \mathbb{R}^n
Math 203, Section 30
Autumn Quarter 2007
Written Exercises from Tuesday, October 2

Exercise 0.0.1 Using only the axioms for an ordered integral domain, prove the following:
Given $a, b \geq 0$, we have $a^2 > b^2$ if and only if $a > b$.

Exercise 0.0.2 Give an example of a ring in which it is not true that $a^2 = 0$ implies $a = 0$.

Exercise 0.0.3 For $n \geq 2$, construct a bilinear form $\langle \cdot, \cdot \rangle : \mathbb{E}^n \times \mathbb{E}^n \rightarrow \mathbb{R}$ that is symmetric and positive (for every $\mathbf{v} \in \mathbb{E}^n$, we have $\langle \mathbf{v}, \mathbf{v} \rangle \geq 0$) but not positive definite (there is some $\mathbf{v} \neq \mathbf{0}$ such that $\langle \mathbf{v}, \mathbf{v} \rangle = 0$).

Exercise 0.0.4 Provide a definition for the function $\cos^{-1} : [-1, 1] \rightarrow \mathbb{R}$ that does not refer to elementary trigonometry or geometric linear algebra.