

Analysis in \mathbb{R}^n
Math 205, Section 30
Spring Quarter 2008
Written Exercises from Week 6

Exercise 0.0.1 Show that any collection of disjoint intervals that covers \mathbb{R} must be countable.

Exercise 0.0.2

- i.* Give a reasonable definition of what it means for $f : A \rightarrow \mathbb{R}$ to be integrable if A is a closed rectangle but f is unbounded. What are the limitations of your definition?
- ii.* Give a reasonable definition of what it means for $f : \mathbb{R}^n \rightarrow \mathbb{R}$ to be integrable if f is bounded. What are the limitations of your definition?

Exercise 0.0.3 This was 0.1.10 on the Fubini script.

- i.* Construct an example of a set $A \subset [0, 1] \times [0, 1]$ such that $\partial A = [0, 1] \times [0, 1]$ but the intersection of A with each horizontal line and each vertical line in \mathbb{R}^2 contains at most one point.
- ii.* If χ_A is the characteristic function of A , show that $\int_{[0,1]} (\int_{[0,1]} \chi_A(x, y) dx) dy = 0$ and $\int_{[0,1]} (\int_{[0,1]} \chi_A(x, y) dy) dx = 0$, but that χ_A is not integrable.