Analysis in $\mathbb{R}^n$
Math 203, Section 30
Autumn Quarter 2007
Written Exercises from Thursday, October 18

Exercise 0.0.1 Let $f \in C[0,1]$ be such that $f(x) \geq 0$ for all $x \in [0,1]$, and suppose there exists $c \in [0,1]$ such that $f(c) > 0$. Show that $\int_0^1 f(x)\,dx > 0$.

Exercise 0.0.2 We showed in class that if $\mathbf{p}_0 = (x_1, x_2, x_3)$ and $\mathbf{v}_0 = (a_1, a_2, a_3)$, then the parametrized equation for the line through $\mathbf{p}_0$ in the direction of $\mathbf{v}_0$ is given by $\mathbf{p}(t) = \mathbf{p}_0 + t\mathbf{v}_0$. We also saw that given the coordinates above, and writing the general point on the line as $\mathbf{p} = (y_1, y_2, y_3)$, the line is also determined by the constraint equations $\frac{y_1 - x_1}{a_1} = \frac{y_2 - x_2}{a_2} = \frac{y_3 - x_3}{a_3}$. However, this last set of equations is valid only when none of the a_i are zero. What are the two constraint equations in the case that one or more of the a_i are zero?

Exercise 0.0.3 Write the equation for a plane in $\mathbb{R}^4$ in two forms:

- parametrized form in terms of given points and vectors
- constraint equations in terms of coordinates

Exercise 0.0.4 Consider the vector space $C[0,1] = \{f : [0,1] \to \mathbb{R} \mid f \text{ is continuous}\}$ with inner product $\langle f, g \rangle = \int_0^1 f(x)g(x)\,dx$. Consider the collection of functions $\{f_n(x) = x^n \mid n \in \mathbb{N} \cup \{0\}\} \subset C[0,1]$. Use the Gram-Schmidt Orthogonalization Process on this collection of vectors to produce an orthonormal set.

Exercise 0.0.5 In the space $C[0,1]$ with inner product as in Exercise 0.0.4, show that the collection of vectors $\{1\} \cup \{f_n(x) = \cos(n\pi x) \mid n \in \mathbb{N}\} \cup \{g_n(x) = \sin(n\pi x) \mid n \in \mathbb{N}\}$ forms an orthogonal set. What would you need to do in order to make it an orthonormal set?