Math 207, Section 31: Honors Analysis I
Autumn Quarter 2009
John Boller
Homework 2, Version 2
Due: Monday, October 12, 2009

1. (*) Read Sally, Chapter 1, especially Sections 7–9.

2. (*) Read Sally, Chapter 3.

3. (*) Read Kolmogorov and Fomin, Chapter 1, especially Section 3.

4. (*) Read Sally, Chapter 2, sometime.

5. Suppose we define an *absolute value* on the rational numbers $\mathbb{Q}$ to be a function $a : \mathbb{Q} \to \mathbb{Q}$ satisfiying:

   - $a(x) \geq 0, \forall x \in \mathbb{Q}$, and $a(x) = 0$ iff $x = 0$
   - $a(x \cdot y) = a(x) \cdot a(y), \forall x, y \in \mathbb{Q}$
   - $a(x + y) \leq a(x) + (y), \forall x, y \in \mathbb{Q}$

   Classify all such functions.

6. Sally, Section 3.5, Exercises (*) 3.5.4, (*) 3.5.5, (*) 3.5.10, (*) 3.5.13, 3.5.15, and 3.5.20.

7. Read Section 3.6 and do Exercise 3.6.26.

8. Read Section 3.7 and prove Theorem 3.7.3.

9. Do the exercises in Sally, Chapter 3, Project 10.3.

10. (*) If you did not take Math 160's, do Project 10.1 in Chapter 3.

11. Read Section 3.1 and do Exercises 3.1.9, (*) 3.1.11, 3.1.12, and (*) 3.1.14.

12. Read Section 3.2 and do Exercises 3.2.4 and (*) 3.2.9.

13. Classify all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$|f(x) - f(y)| = |x - y|$$

   where $| \cdot |$ is the usual absolute value.