Math 207, Section 31: Honors Analysis I
Autumn Quarter 2009
John Boller
Homework 3, Version 2
Due: Monday, October 19, 2009

1. (*) Read Kolmogorov and Fomin, Chapter 2, especially Sections 5 and 6.
2. (*) Read Sally, Chapter 2.
3. (*) Read Sally, Chapter 4.
4. Determine $\mathrm{Aut}_{\mathbb{R}}(\mathbb{C})$, the collection of field automorphisms of $\mathbb{C}$ which fix $\mathbb{R}$ pointwise.
5. Sally, Section 4.2, Exercises (*)4.2.4, (*)4.2.5, 4.2.8, 4.2.9, and 4.2.10.
6. Sally, Section 4.3, Exercises 4.3.5, 4.3.6, (*)4.3.11, (*)4.3.14, (*)4.3.21, 4.3.24, 4.3.25, (*)4.3.26, 4.3.27, 4.3.28, 4.3.29, (*)4.3.32, 4.3.33, (*)4.3.35, 4.3.38, 4.3.40, 4.3.42.
7. If we denote the closure of a set A in a metric space by the notation $\overline{A}$, prove the following:
   (a) If $A \subset B$, then $\overline{A} \subset \overline{B}$.
   (b) $\overline{\overline{A}} = \overline{A}$
   (c) $\overline{A \cup B} = \overline{A} \cup \overline{B}$
8. Do Sally, Project 4.6.1, on General Point Set Topology, parts 1 (Basic Notions) and 2 (Separation Properties).
9. Minkowski's Inequality states that for $p \geq 1$:
   If $a = (a_1, \ldots, a_n)$ and $b = (b_1, \ldots, b_n)$ are in $\mathbb{R}^n$, then
$$\left(\sum_{k=1}^{n} |a_k + b_k|^p\right)^{1/p} \leq \left(\sum_{k=1}^{n} |a_k|^p\right)^{1/p} + \left(\sum_{k=1}^{n} |b_k|^p\right)^{1/p}.$$
   (a) Prove Minkowski's Inequality.
   (b) Show that Minkowski's Inequaltiy is false for $0 < p < 1$.
10. Let $X = \mathbb{R}^n$ with the usual metric. Show that the only subsets of X that are both open and closed are $\emptyset$ and X itself.