Math 208, Section 31: Honors Analysis II
Winter Quarter 2010
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Homework 3, Final Version
Due: Monday, January 25, 2010

1. (*) Read a good source on the Inverse and Implicit Function Theorems. Perhaps the BS notes, Rudin (Chapter 9), Edwards (Chapter 2), or Lang (Chapter 15).

2. Let V be a real vector space. A norm on V is a map $||\cdot|| : V \to \mathbb{R}$ satisfying:

   (a) $||x|| \geq 0, \forall x \in V$ and $||x|| = 0$ iff $x = 0$

   (b) $||cx|| = |c|||x||, \forall c \in \mathbb{R}, \forall x \in V$

   (c) $||x + y|| \leq ||x|| + ||y||, \forall x, y \in V$

   Two norms $||\cdot||_1$ and $||\cdot||_2$ on V are said to be equivalent if there exist real scalars $\alpha_1, \alpha_2 > 0$ such that $\alpha_1||x||_1 \leq ||x||_2 \leq \alpha_2||x||_1$ for all $x \in V$.

   (a) Show that any two norms on $\mathbb{R}^n$ are equivalent.

   (b) Show that there exist inequivalent norms on $\mathbb{R}^{\mathbb{N}} = \{(x_1, x_2, \dots) \mid x_i \in \mathbb{R}, \forall i \in \mathbb{N}\}$.

3. Read Section 1.5 in the BS notes on Taylor's Theorem, and do Exercises (*) 1.5.4, 1.5.5, (*) 1.5.6, (*) 1.5.7, 1.5.10(i), and 1.5.11, and prove Taylor's Theorem 1.5.8.

4. Read Section 1.6 in the BS notes on Tangent Hyperplanes, and do Exercises (*) 1.6.2, 1.6.6, (*) 1.6.7, 1.6.8, and 1.6.9.

5. Read Section 1.7 in the BS notes on Max/Min Problems, and do Exercises (*) 1.7.5, 1.7.6, 1.7.7, and 1.7.8.

6. Read Section 1.8 in the BS notes on Lagrange Multipliers, and do Exercises (*) 1.8.5, 1.8.6, 1.8.7, 1.8.13, and (*) 1.8.14.