Math 207, Section 31: Honors Analysis I
Autumn Quarter 2009
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Homework 5, Final Version
Due: Monday, November 2, 2009

1. (*) Read Kolmogorov and Fomin, Chapter 2, especially Sections 5–7.
2. (*) Read Kolmogorov and Fomin, Chapter 3, especially Sections 9–11.
3. (*) Read Sally, Chapter 4.
4. Sally, Section 4.4, Exercises (*) 4.4.3, (*) 4.4.4, 4.4.16, (*) 4.4.18, 4.4.21, 4.4.26, 4.4.34, 4.4.36, and (*) 4.4.37.
5. Sally, Section 4.5, Exercises 4.5.9, 4.5.10, 4.5.11, 4.5.18, 4.5.20, (*) 4.5.27, (*) 4.5.28, (*) 4.5.31, 4.5.35, and 4.5.39.
6. Do Sally, Project 4.6.1, on General Point Set Topology, parts 3 (New Topologies from Old) and 4 (Compactness).
7. (*) Read Sally, Project 2.6.1, on Groups.
8. Let $\ell^\infty = \{(x_n)_{n=1}^\infty \in \mathbb{R}^\mathbb{N} \mid \sup_n |x_n| < \infty\}$ have the sup norm $||x||_\infty = \sup_{n\in\mathbb{N}} |x_n|$. If 0 represents the sequence of all zeroes in ℓ^∞, show that $\overline{B_1(0)}$ is not compact.
9. Show that every metric space (X, d) has a *completion* which is unique up to isometry. That is, show that there exists a complete metric space (X^*, d^*) and an injection $i : X \to X^*$ such that $i(X)$ is dense in X^* and $d^*(i(x), i(y)) = d(x, y)$ for all $x, y \in X$. Show that if X^{**} is another such completion, then there is an isometry between X^* and X^{**}.
10. Let $C[a, b] = \{f : [a, b] \to \mathbb{R} \mid f \text{ is continuous}\}$. Show that the map

$$||f||_2 = \left(\int_a^b |f(x)|^2 \, dx \right)^{1/2}$$

defines a norm on $C[a, b]$.