Math 208, Section 31: Honors Analysis II
Winter Quarter 2010
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Homework 4, Final Version
Due: Monday, February 8, 2010

1. (*) Read a good source on the Riemann Integration in $\mathbb{R}^n$. Perhaps the BS notes, volume II, Apostol (Chapter 14), Edwards (Chapter 4), or Lang (Chapter 20).

2. **Theorem (Hadamard)** Let $X = [x_{ij}] \in M_n\mathbb{R}$. For each $i = 1, \ldots, n$, let $X_i = (x_{i1}, \ldots, x_{in})$ be the i-th row of X, and let $d_i = ||X_i||$. If $\Delta = \det(X)$, then $|\Delta| \leq d_1 \cdots d_n$.

   Prove this theorem using Lagrange's method by considering Δ as a function of n^2 variables subject to n constraints. (Hint: Show that at the extrema of Δ, we have $\Delta^2 = \det(D)$, where D is the diagonal matrix with diagonal entries $d_1^2, \ldots, d_n^2$.)

3. Let $A \in M_n(\mathbb{R})$ by symmetric, and suppose $\det(A) \neq 0$. Let $q : \mathbb{R}^n \to \mathbb{R}$ be the quadratic form defined by $q(x) = x^t Ax$. For each $k = 1, \ldots, n$, let $\Delta_k = \det(A_k)$, where A_k is the $k \times k$ submatrix of A obtained by deleting the last $n - k$ rows and the last $n - k$ columns of A.

   (a) Show that q is positive-definite if and only if $\Delta_k > 0$ for each $k = 1, \ldots, n$.

   (b) Show that q is negative-definite if and only if $(-1)^k\Delta_k > 0$ for each $k = 1, \ldots, n$.

4. Use our tools of optimization to classify the local extrema of the functions:

   (a) $f(x, y) = x + x^2 + xy + y^3$

   (b) $g(x, y) = (x - 1)^4 + (x - y)^4$

5. State and prove a version of the Spectral Theorem for elements of $M_n(\mathbb{C})$.

6. (*) Read Sections 2.1–2.4 in the BS notes II and do all the exercises.

7. Read Section 2.5 in the BS notes II and do Exercises (*) 2.5.5, (*) 2.5.6, 2.5.8, 2.5.10, (*) 2.5.12, (*) 2.5.13, and 2.5.16.

8. Read Section 2.6 in the BS notes II, prove Theorems 2.6.1, 2.6.2, and 2.6.3 and Propositions (*) 2.6.4 and (*) 2.6.5.

9. Read Section 2.7 in the BS notes II and do Exercises (*) 2.7.2, (*) 2.7.3, (*) 2.7.5, 2.7.7, 2.7.8, and (*) 2.7.11.

10. (*) Re-do any midterm problems you did not solve correctly the first time!