Math 259, Section 33: Honors Algebra III
Spring Quarter 2009
John Boller
Homework 4, Version 2 (one minor correction to # 6)
Due: Friday, May 1, 2009

1. (*) Read Dummit and Foote, Chapter 13.
2. Dummit and Foote, Section 13.2, #15.
3. Dummit and Foote, Section 13.2, #17.
4. Dummit and Foote, Section 13.2, #18.
5. Dummit and Foote, Section 13.3, #1:

   Show that it is impossible to construct a regular 9-gon using straightedge and compass.
6. (*) Dummit and Foote, Section 13.3, #4:

   To construct the regular 7-gon, it is necessary and sufficient to construct the length $\cos(2\pi/7)$. Show that $\alpha = 2\cos(2\pi/7)$ satisfies the equation $x^3 + x^2 - 2x - 1 = 0$ and that therefore the the regular 7-gon is not constructible.
7. Dummit and Foote, Section 13.3, #5:

   Use the fact that $\alpha = 2\cos(2\pi/5)$ satisfies the equation $x^2 + x - 1 = 0$ to conclude that the regular 5-gon is construcible.
8. Dummit and Foote, Section 13.4, #2,4:

   (a) Determine the splitting field and its degree over $\mathbb{Q}$ of $x^6 - 4$.

   (b) Determine the splitting field and its degree over $\mathbb{Q}$ of $x^4 + 2$.
9. Prove that $f(x) \in F[x]$ is separable if and only if it is relatively prime to its formal derivative, that is $(f(x), f'(x)) = 1$.
10. Dummit and Foote, Section 13.5, # 5:

    For any prime p and any nonzero $a \in \mathbb{F}_p$, prove that $x^p - x + a$ is irreducible and separable over $\mathbb{F}_p$.
11. (*) Dummit and Foote, Section 13.5, # 1–4, 6–10.