1. (*) Read Dummit and Foote, Sections 14.3–14.7.

2. (*) Dummit and Foote, Section 14.3, #1–7 and 10.

3. Dummit and Foote, Section 14.3, #8:
   Determine the splitting field of the polynomial \( f(x) = x^p - x - a \) over \( \mathbb{F}_p \), where \( a \in \mathbb{F}_p \) and \( a \neq 0 \).
   Show explicitly that the Galois group is cyclic.

4. Dummit and Foote, Section 14.3, #11:
   Prove that \( f(x) = x^{p^n} - x + 1 \) is irreducible over \( \mathbb{F}_p \) only when \( n = 1 \) or \( n = p = 2 \).

5. Let \( K = \mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5}) \). Find an element \( \alpha \in K \) such that \( K = \mathbb{Q}(\alpha) \).

6. Dummit and Foote, Section 14.4, #6:
   Read Proposition 24 and prove that \( \mathbb{F}_p(x, y)/\mathbb{F}_p(x^p, y^p) \) is not a simple extension by explicitly exhibiting an infinite number of intermediate subfields.

7. Determine the Galois group \( \text{Gal}(\mathbb{Q}(\zeta_n)/\mathbb{Q}) \) and draw lattice diagrams of the intermediate subfields for \( n = 13 \) and \( n = 16 \).

8. Find the other roots of Cardano’s two cubics:
   - \( x^3 - 15x - 4 \)
   - \( x^3 + 6x - 20 \)

9. (*) Dummit and Foote, Section 14.6. #1–14. (Please try some of these!)

10. Dummit and Foote, Section 14.6. #15:
   Let \( p \in \mathbb{Z} \) be prime, and let \( f(x) = x^4 + px + p \in \mathbb{Q}[x] \).
   (a) Show that \( f(x) \) is irreducible for every prime \( p \).
   (b) Show that if \( p \neq 3, 5 \), then the Galois group of \( f(x) \) is \( S_4 \).
   (c) Show that if \( p = 3 \), then the Galois group of \( f(x) \) is \( D_4 \).
   (d) Show that if \( p = 5 \), then the Galois group of \( f(x) \) is \( C_4 \).