## EXAM 2 REVIEW

Here is a list of things you should know for the second 153 exam, by section.

10.1) You should know about upper and lower bounds, least upper bounds and greatest lower bounds, and the least upper bound axiom. You should know that you can find elements of a set arbitrarily close to the least upper bound or greatest lower bound (10.1.2 and 10.1.4).

10.2) You should know the definition of: sequence, boundedness of a sequence, increasing, decreasing, nonincreasing, nondecreasing, monotonic. If you've been having trouble with the idea of finding a formula for a sequence in terms of $n$, try problems 1-8 in this section. (Number 8 is pretty challenging, I think–don't worry too much if you can't figure it out.)

10.3) Know the definition of a limit–you will almost surely have to use this on the exam to prove a limit exists. Know the definition of convergent and divergent. Be able to prove that every convergent sequence is bounded (10.3.4). Also be able to prove 10.3.7 parts (i) and (ii). Know the squeeze theorem (10.3.9). Know theorem 10.3.12 for limits and continuous functions. (Although you won't need to prove it–we didn't prove it in class–you should understand why it's true and how to use it.)

10.4) You should be able to look at these and know what the limits are without too much thinking. Be able to prove the more common ones: $\lim x^{1/n}, \lim x^n$, and $\lim(1 + x/n)^n$.

10.5, 10.6) Be able to use L'Hôpital's rule. You don't need to prove it (or know the Cauchy Mean Value Theorem), but you need to know when you are allowed to use it and when you aren't. Know how to deal with all of the other indeterminate forms $0^0, 1^\infty, \infty^0, \infty - \infty$, etc.).

10.7) Know how to do both kinds of improper integrals. If $\pm\infty$ is one of the limits of integration, you have to take a limit; if it's on both sides, you have to take two limits. If your function is unbounded at a point, you may have to take limits from either side to get the integral. Know the comparison test: 10.7.2.

11.1) Know the definition of an infinite series partial sums, what a geometric series is, and the Basic Divergence Test (11.1.6).

You will almost certainly have to prove that a limit exists with the $\varepsilon - N$ definition. In addition, you may have to prove one of these results:

-The limit of a sum of two sequences is the sum of the limits of the sequences (if they exist). (10.3.7.i) -The limit of a sequence times a real number is the limit times that real number (10.3.7.ii). -A convergent sequence is bounded (10.3.4) -You can get as close as possible to a least upper bound (10.1.2).