Maximum Drawdown and Directional Trading

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Abstract: Maximum drawdown (MDD) measures the worst loss of the investor who follows a simple trading strategy of buying and subsequently selling a given asset within a certain time framework. MDD is becoming a popular performance measure since it can capture the size of the market drops. One can view the maximum drawdown as a contingent claim, and price and hedge it accordingly as a derivative contract. Similar contracts can be written on the maximum drawup (MDU). We show that buying a contract on MDD or MDU is equivalent to adopting a momentum trading strategy, while selling it corresponds to contrarian trading. Momentum and contrarian traders represent a larger group of directional traders. We also discuss how pricing and hedging MDD contract can be used to measure the severity of the market crashes, and how MDD influences volatility evolution during the periods of market shocks.

Management of Extreme Financial Events

Suppose that we have an underlying asset whose price process at time t is given by S_t. For example, it could be a stock price, index, interest rate or exchange rate. Denote by M_t its running maximum up to time t:

$$M_t = \max_{u \in [0,t]} S_u.$$

Drawdown D_t is defined as the drop of the asset price from its running maximum:

$$D_t = M_t - S_t. \tag{1}$$

Maximum drawdown MDD_t is defined as the maximal drop of the asset price from its running maximum over a given period of time:

$$MDD_t = \max_{u \in [0,t]} D_u. \tag{2}$$

Average drawdown ADD_t is given by

$$ADD_t = \frac{1}{t}\int_0^t D_u du. \tag{3}$$

Similarly, we can define the concept of drawup, maximum drawup and average drawup. *Drawup* U_t is defined as the increase of the asset price from its running minimum:

$$U_t = S_t - m_t, \tag{4}$$

where

$$m_t = \min_{u\in[0,t]} S_u.$$

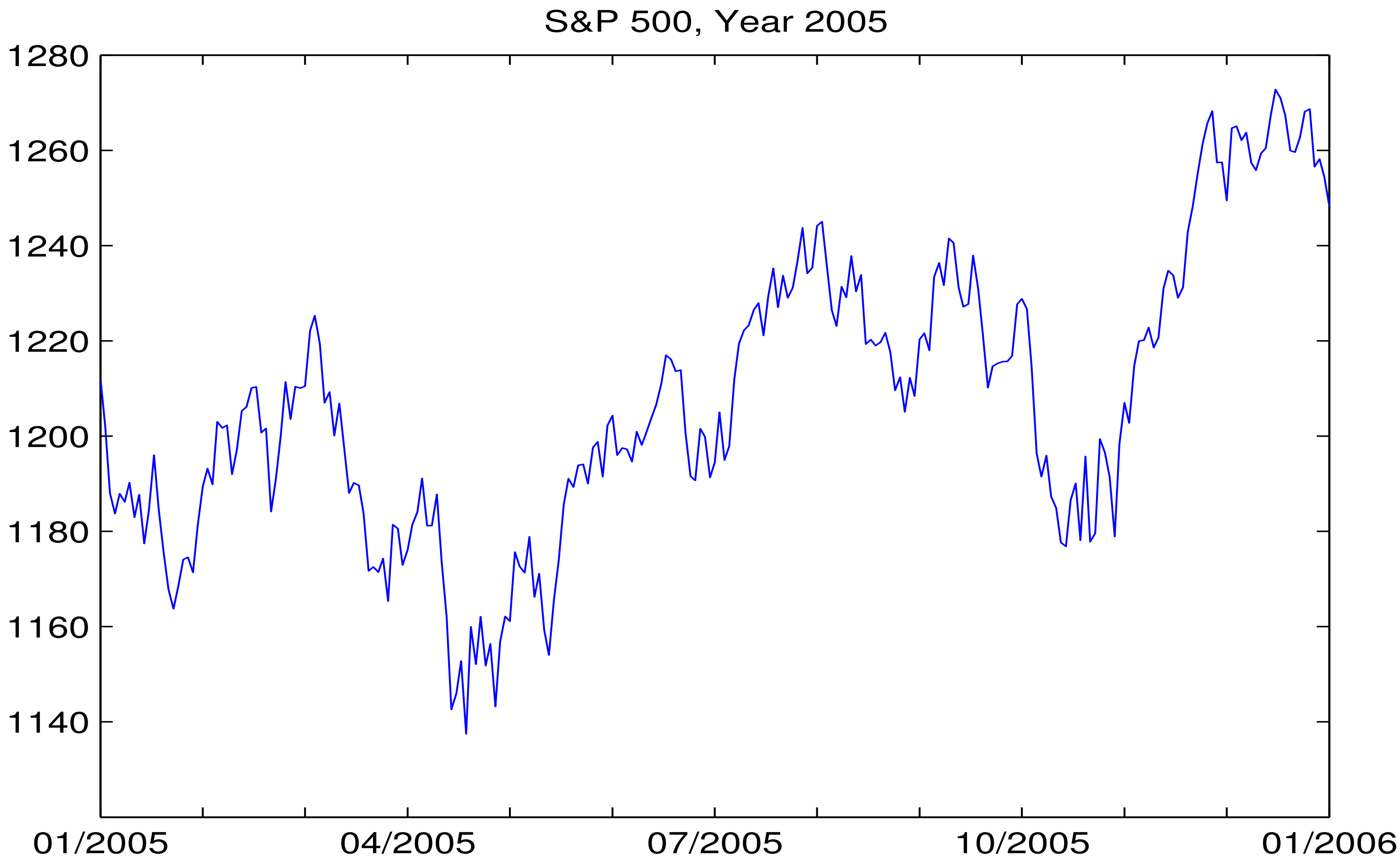
Maximum drawup MDU_t is given by

$$MDU_t = \max_{u\in[0,t]} U_u, \tag{5}$$

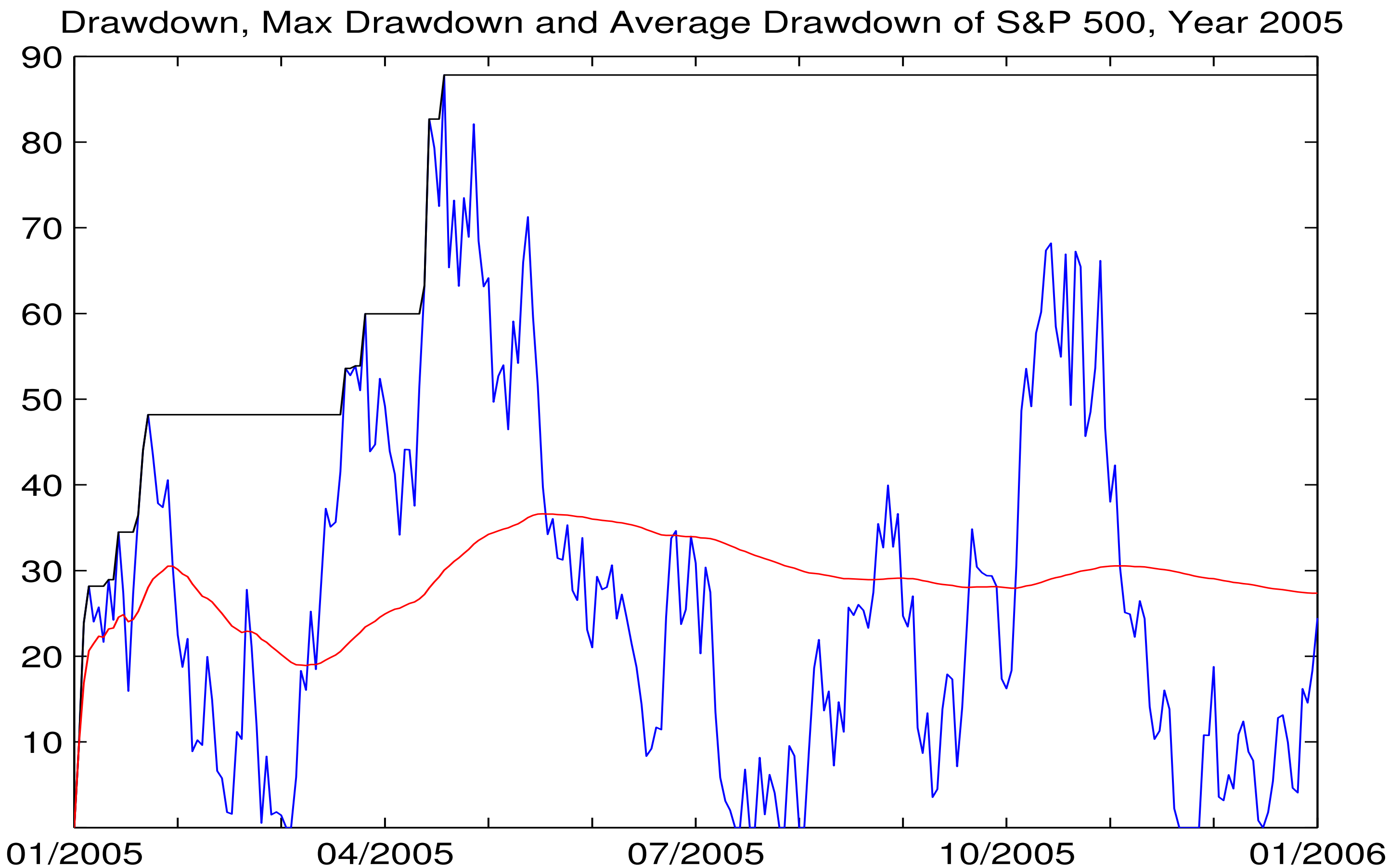
and *Average drawup* ADU_t is defined as

$$ADU_t = \frac{1}{t}\int_0^t U_u du. \tag{6}$$

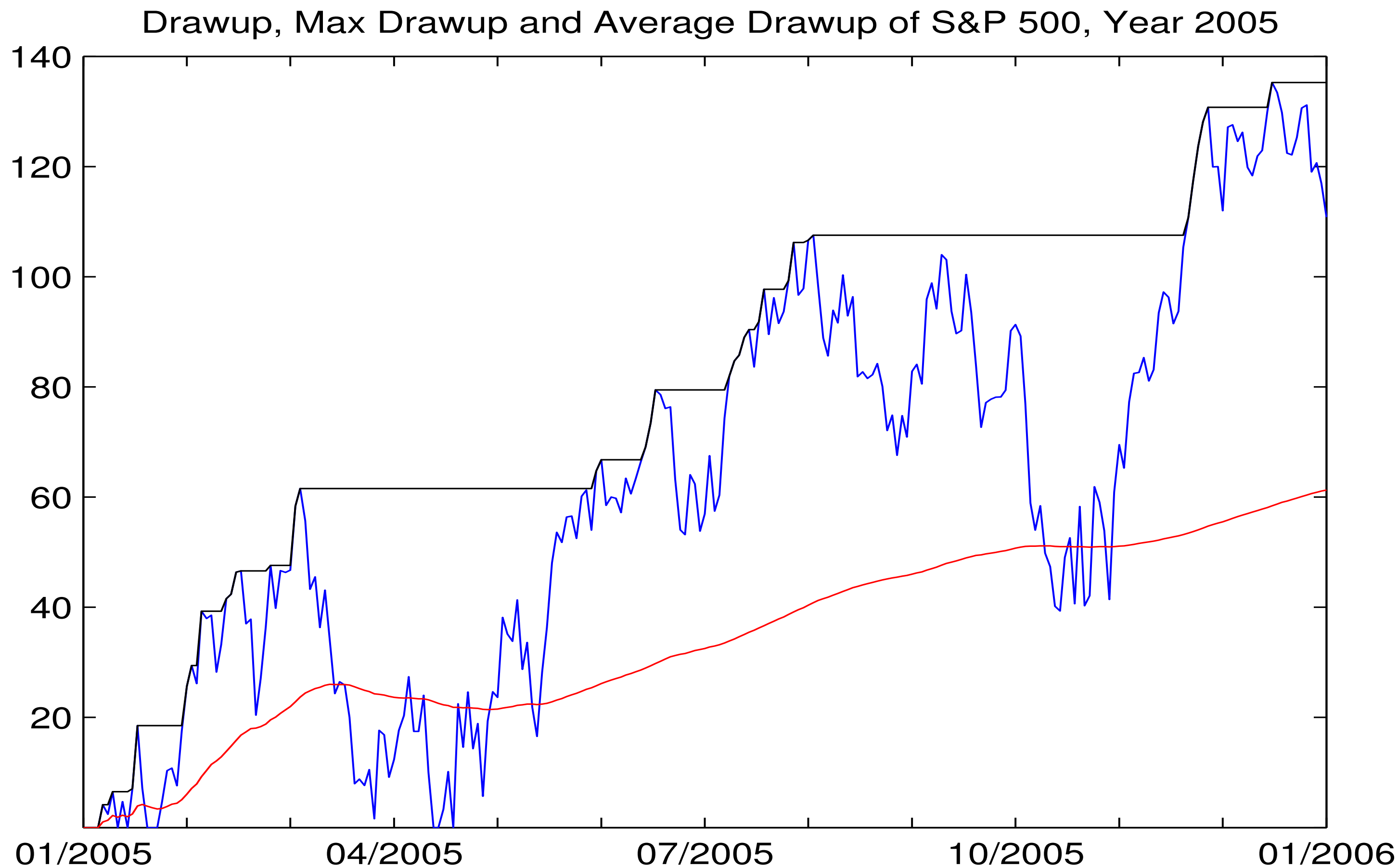
Figures 1–3 illustrate the concepts of the drawdown and the drawup on data taken from S&P500 for the year 2005. Figures 4–5 show the historical evolution of S&P500 and its maximum drawdown for the period starting in 1970.



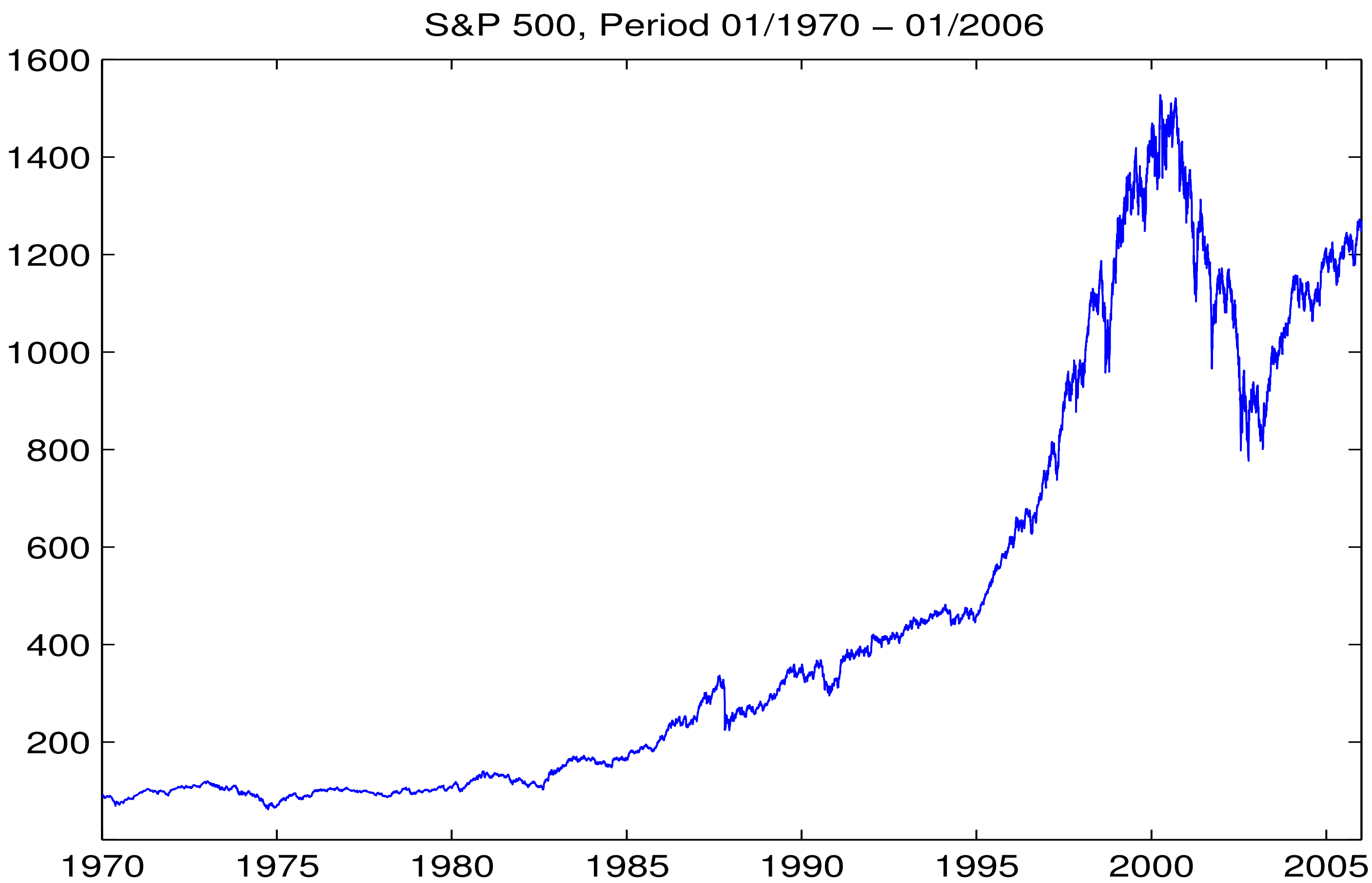
S&P500 daily closing values in 2005.



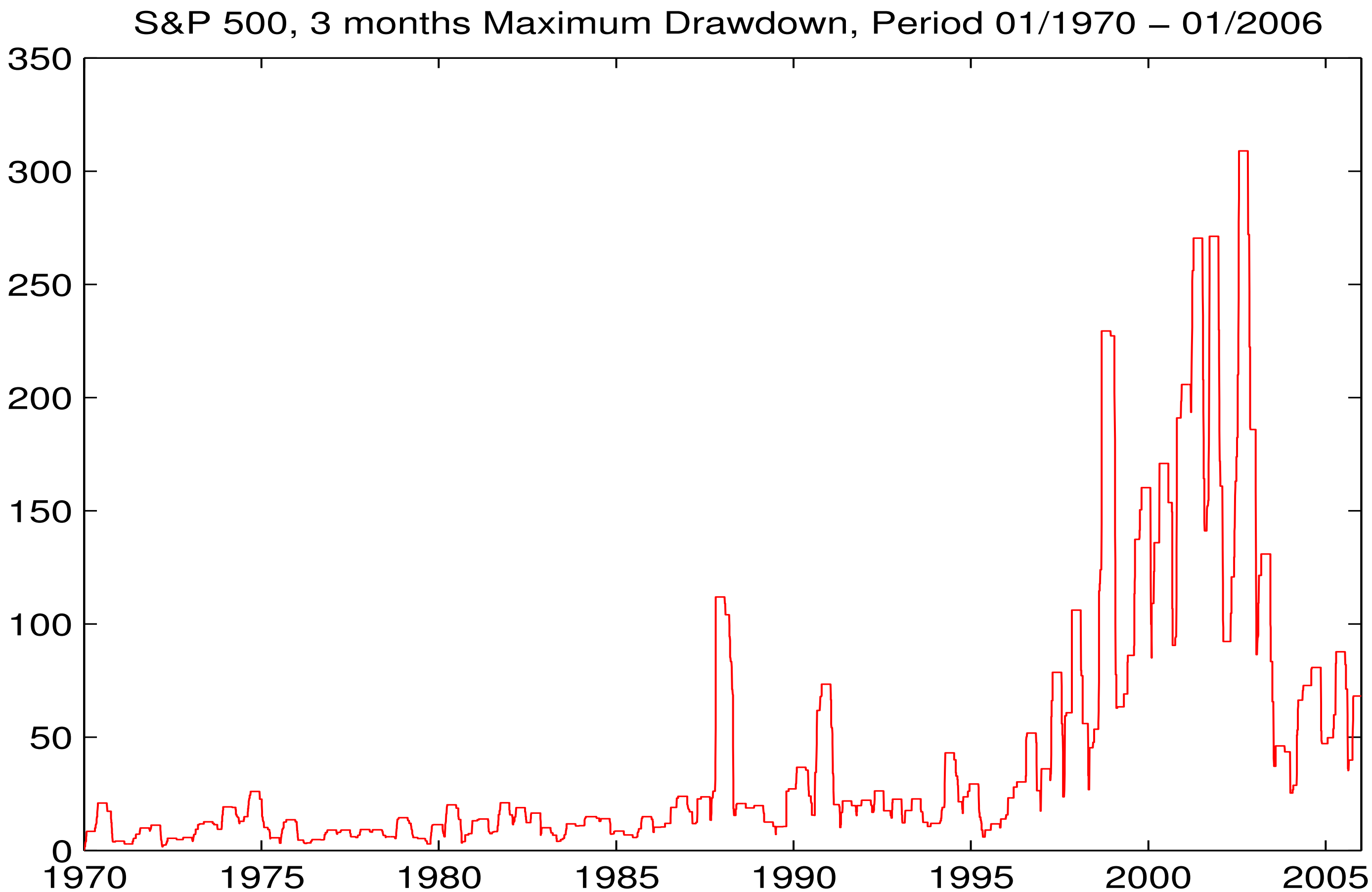
Drawdown (blue), maximum drawdown (black), and average drawdown (red) of S&P500 in 2005.



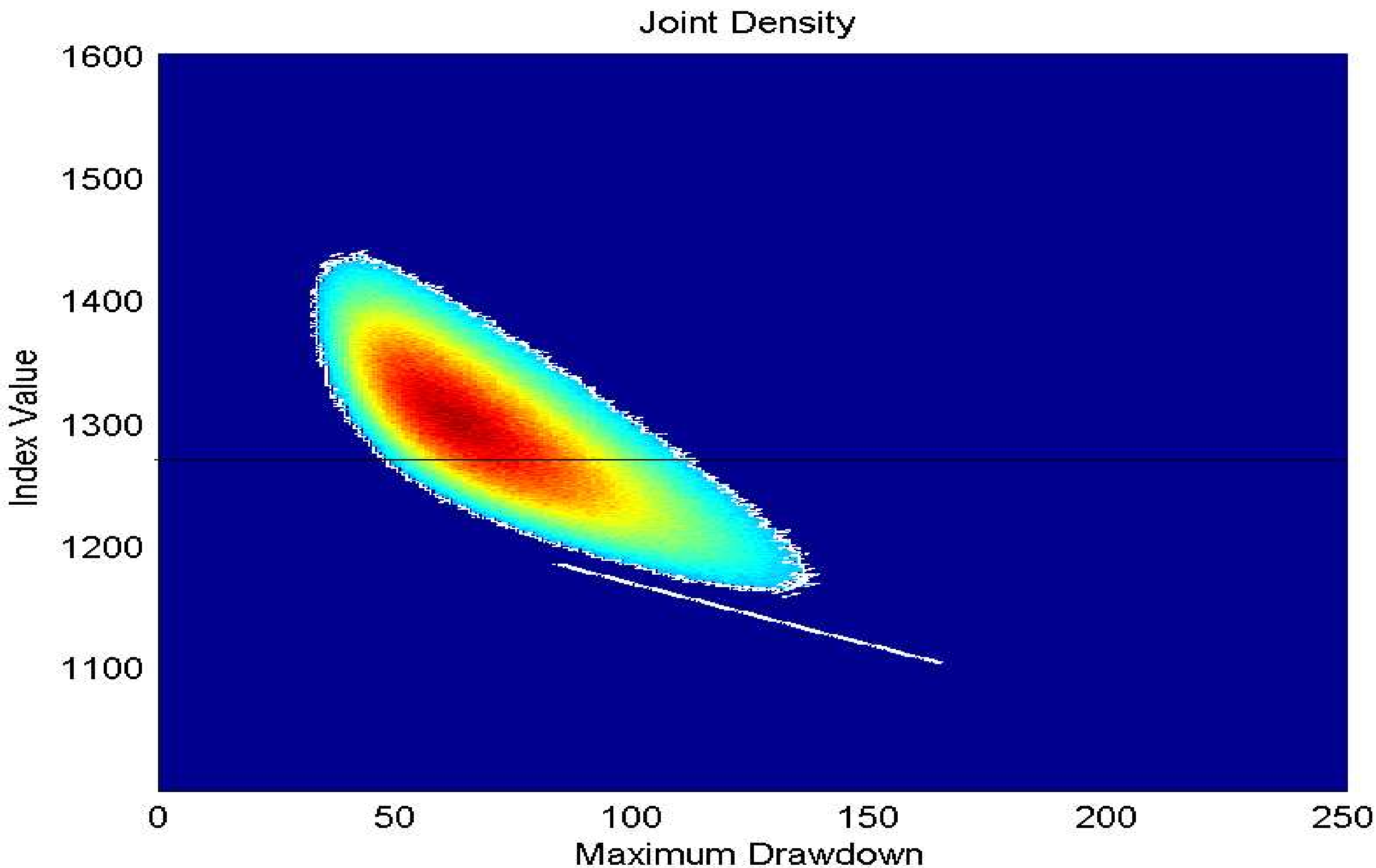
Drawup (blue), maximum drawup (black), and average drawup (red) of S&P500 in 2005.



Index S&P 500 from 01/1970 to 12/2005.



Maximum drawdown taken over 3 months interval, S&P500 index, period from 01/1970 to 12/2005.



3M joint distribution of the S&P 500 and its MDD.
Values from June 30th, 2006 (index 1270.20, VIX 13%).

Why Maximum Drawdown (as Opposed to Puts?)

- You may have a market crash after a market bubble. The put option might still not be in the money.

- Puts lose value when the market is going up. Maximum Drawdown keeps the same value when the market is resetting new maximum ($v_m = 0$).

- Maximum Drawdown is important for directional traders. In particular *Momentum traders* believe that the current trends will continue (essentially betting that the realized drawdown, drawup, or range will be higher than expected). *Contrarian* *traders* believe the opposite, that the trends will revert (betting that the realized drawdown, drawup, or range will be smaller than expected. Traditionally, the trends have been determined from regression analysis of the market data. The link between the MDD and directional traders gives an alternative approach, namely adopt a hedging strategy for the MDD contract.

- The price of a futures contract on maximum drawdown or average drawdown can serve as an important risk measure indicator which would be quoted by the market (rather than determined internally). When the market is in a bubble, it is reasonable to expect that the prices of drawdown contracts would be significantly higher. On the other hand, when the market is stable, or when it exhibits mean reversion behavior, the prices of drawdown contracts would become cheaper.

Pricing

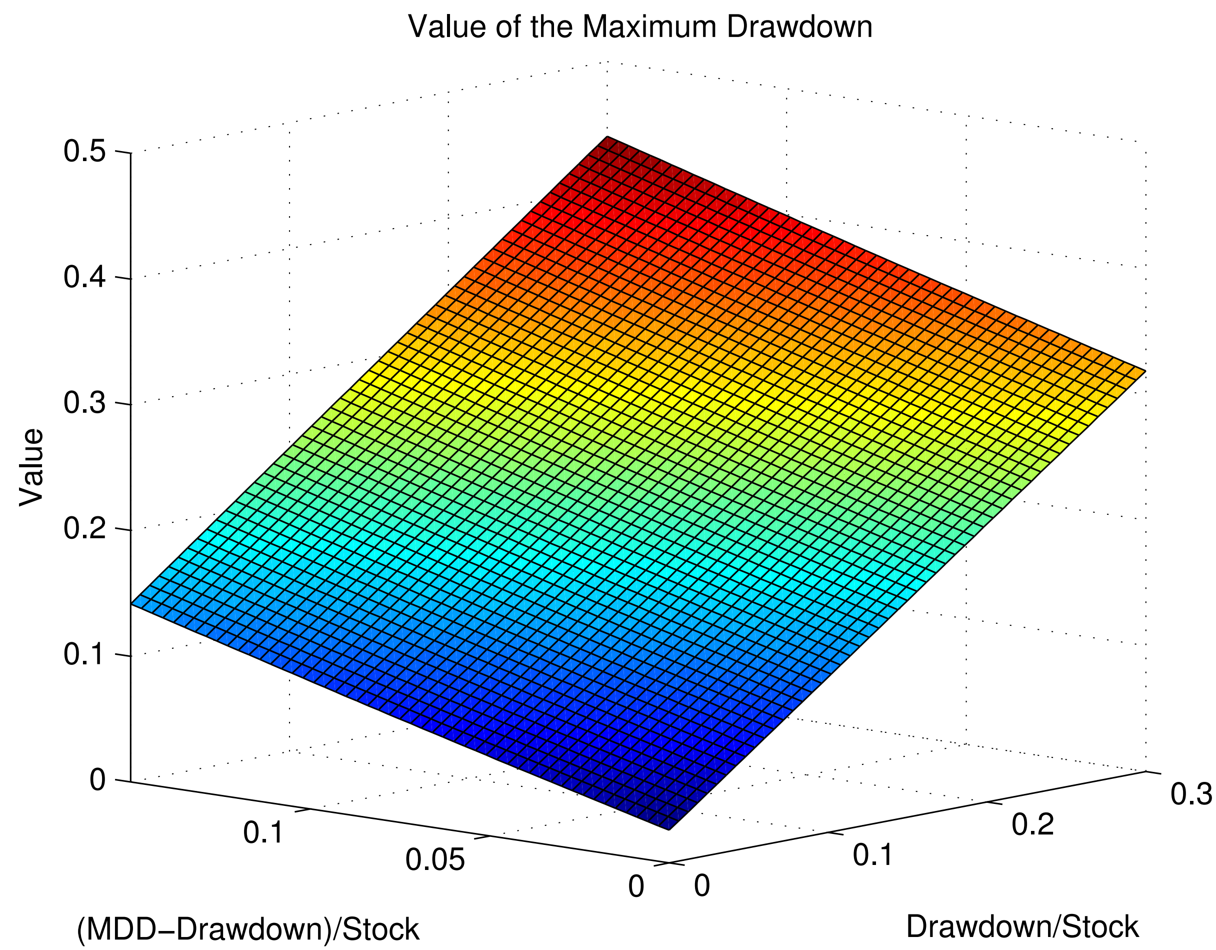
As for the pricing, the value $v(t, S_t, M_t, MDD_t)$ of any type of contract depending on the maximum drawdown is given by taking the conditional expectation of the discounted payoff under the risk neutral measure:

$$v(t, S_t, M_t, MDD_t) = \mathbb{E}[e^{-r(T-t)} f(\{MDD_u\}_{u=0}^{T}) | S_t, M_t, MDD_t].$$

A similar formula applies for the pricing of contracts that depend on the average drawdown, ADD_t. Here, the function f determines the type of payoff defined by the contract (for instance, $f(\{MDD_t\}_{t=0}^{T}) = MDD_T - K$ for the forward contract, etc.). For the evolution of the underlying asset under the risk neutral measure, we may assume that

$$dS_t = rS_t dt + g(t, S_t) dN_t,$$

for a general martingale N_t (diffusion or jump type process). Other possible evolutions, such as a mean reversion type process, could be considered for the asset dynamics of S_t.



Value Function.

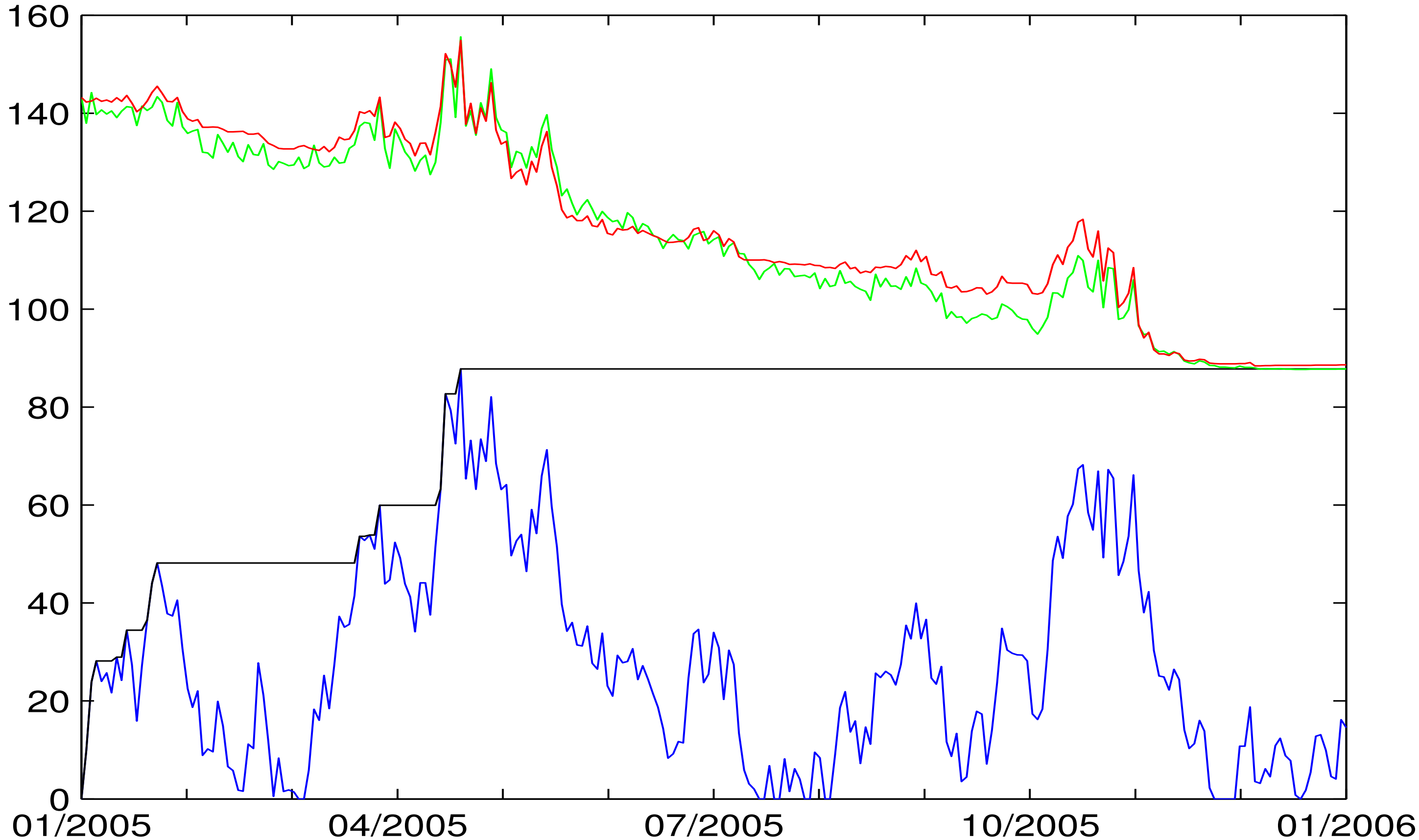
Corresponding Hedge.

Example of Hedging: Maximum Drawdown

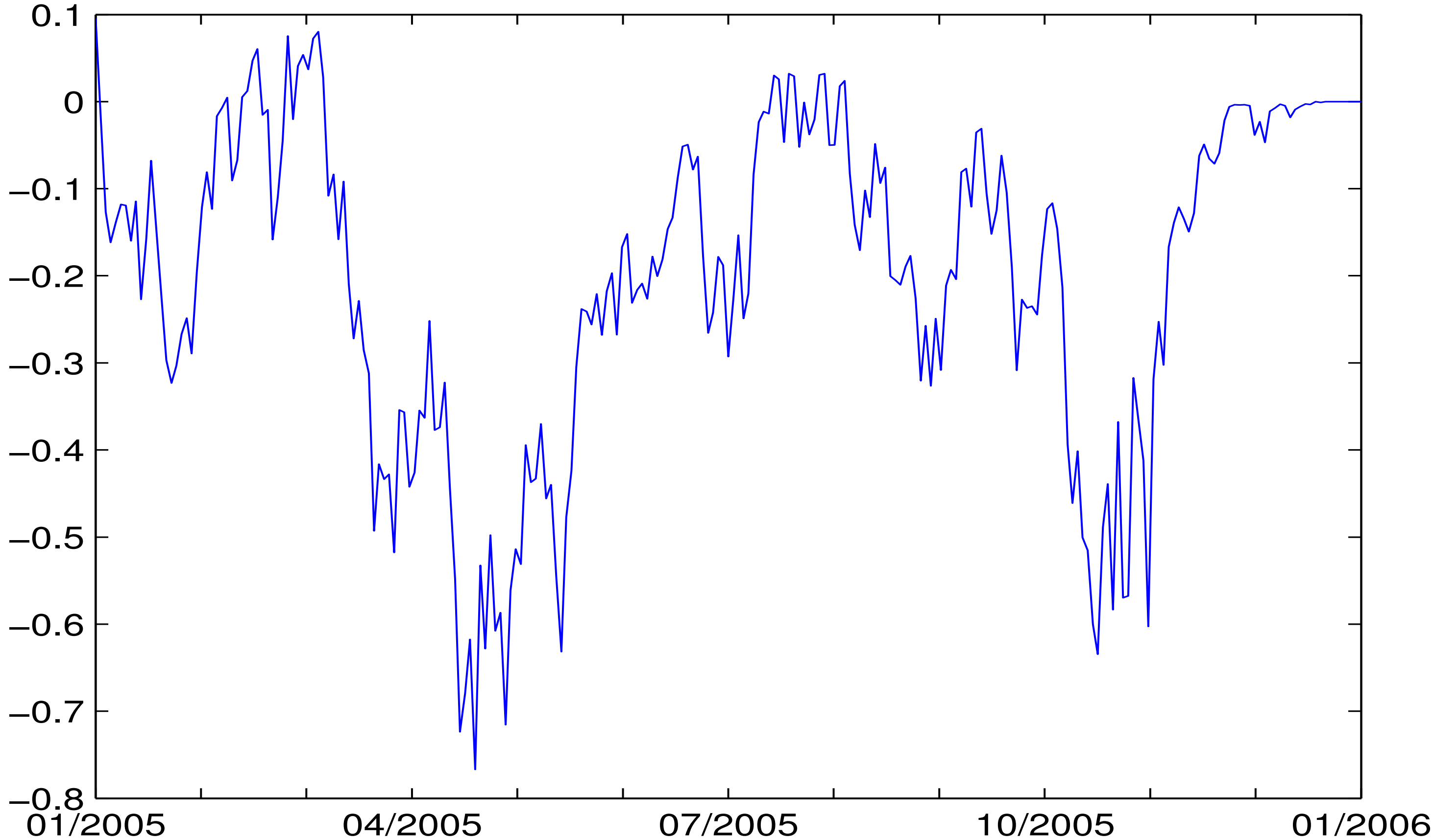
The hedge of the contract is given by the standard delta hedge

$$\Delta(t, s, m, mdd) = v_s(t, s, m, mdd)$$

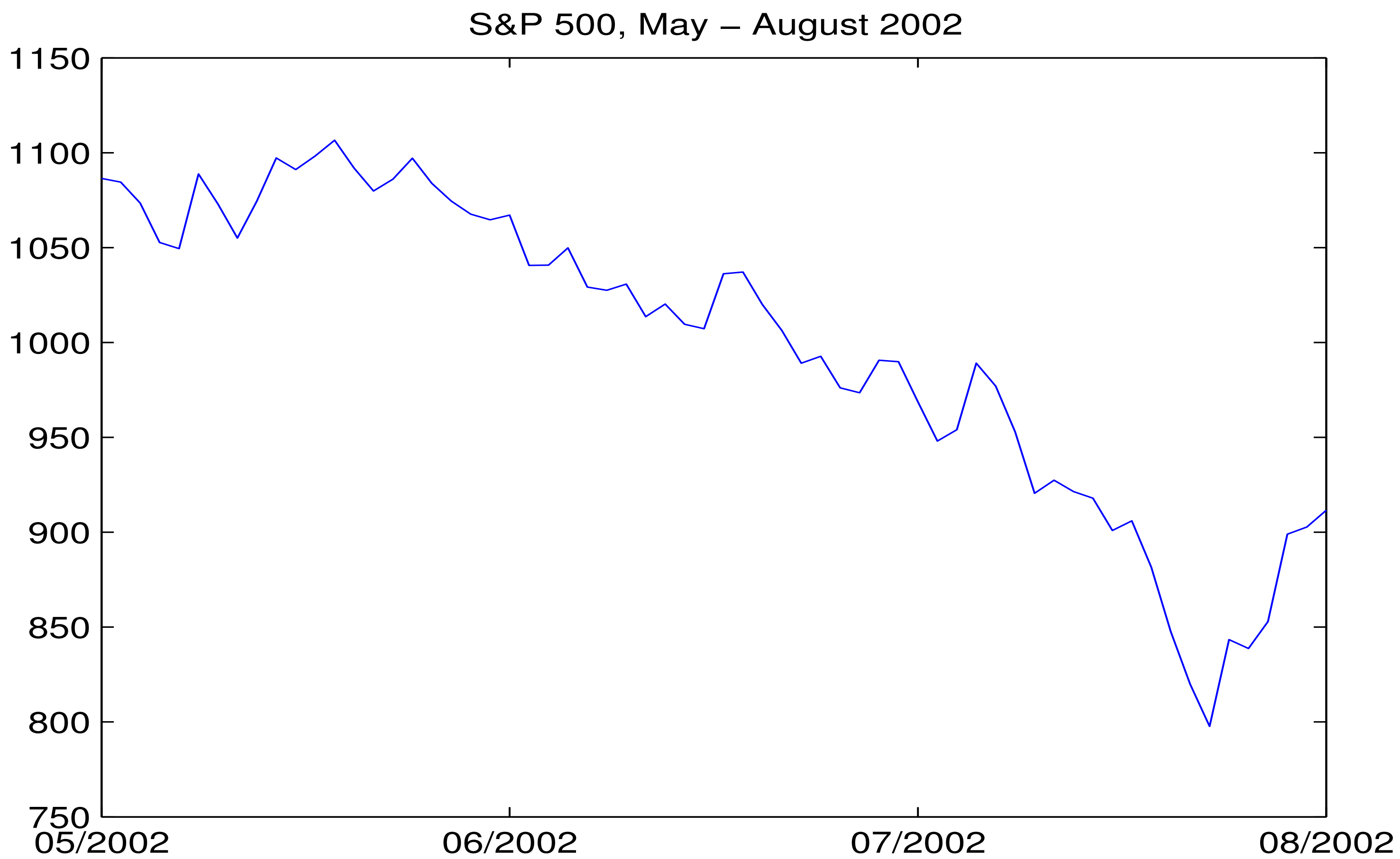
We present two case studies: hedging MDD in 2005 (entire year), and hedging the worst historical 3 months drop (May - July 2002).



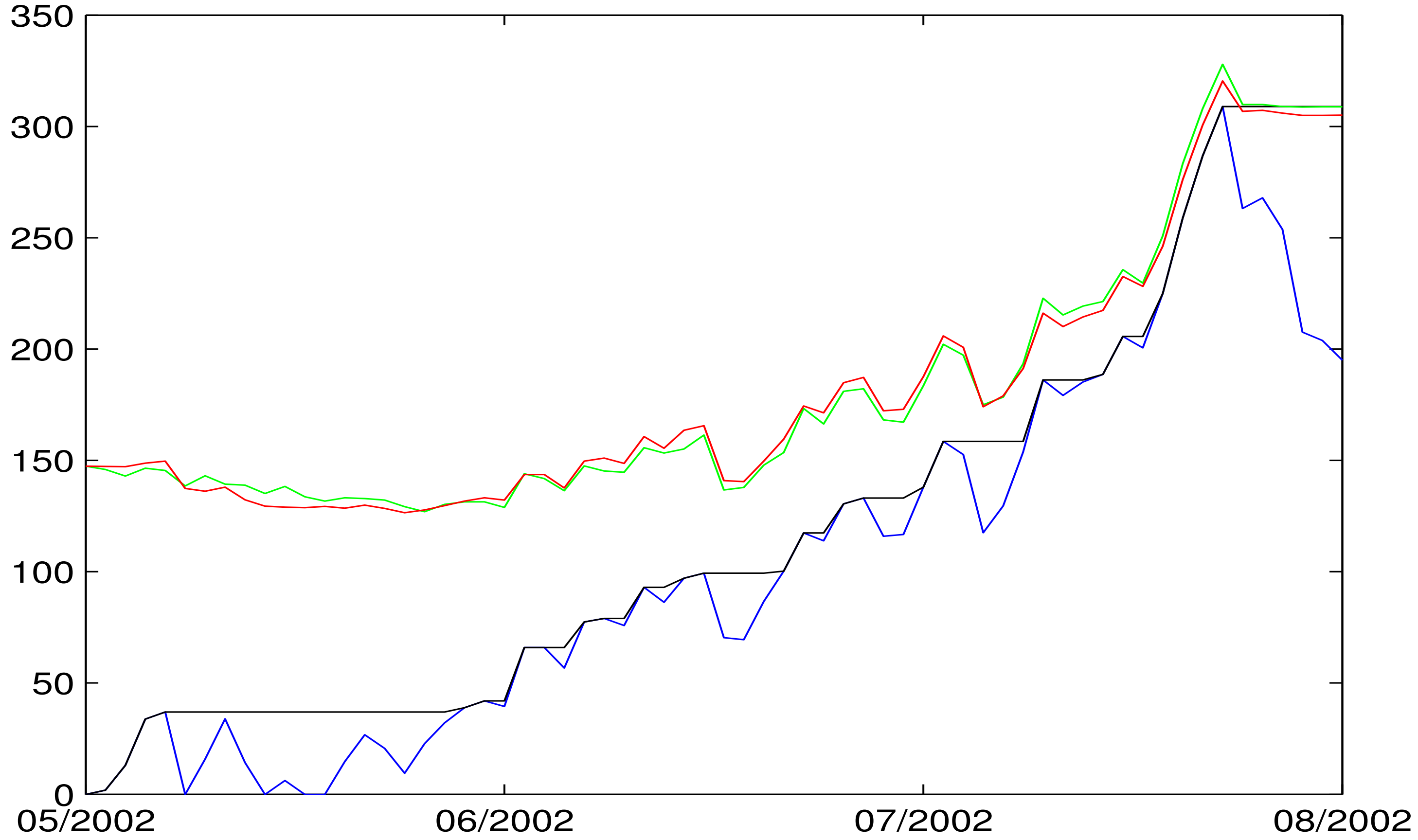
DD (blue), MDD (black), MDD Value (green), Replicating Portfolio (red).
Opening value 1,211.92, interest rate $r = 0.03$, volatility $\sigma = 11\%$.



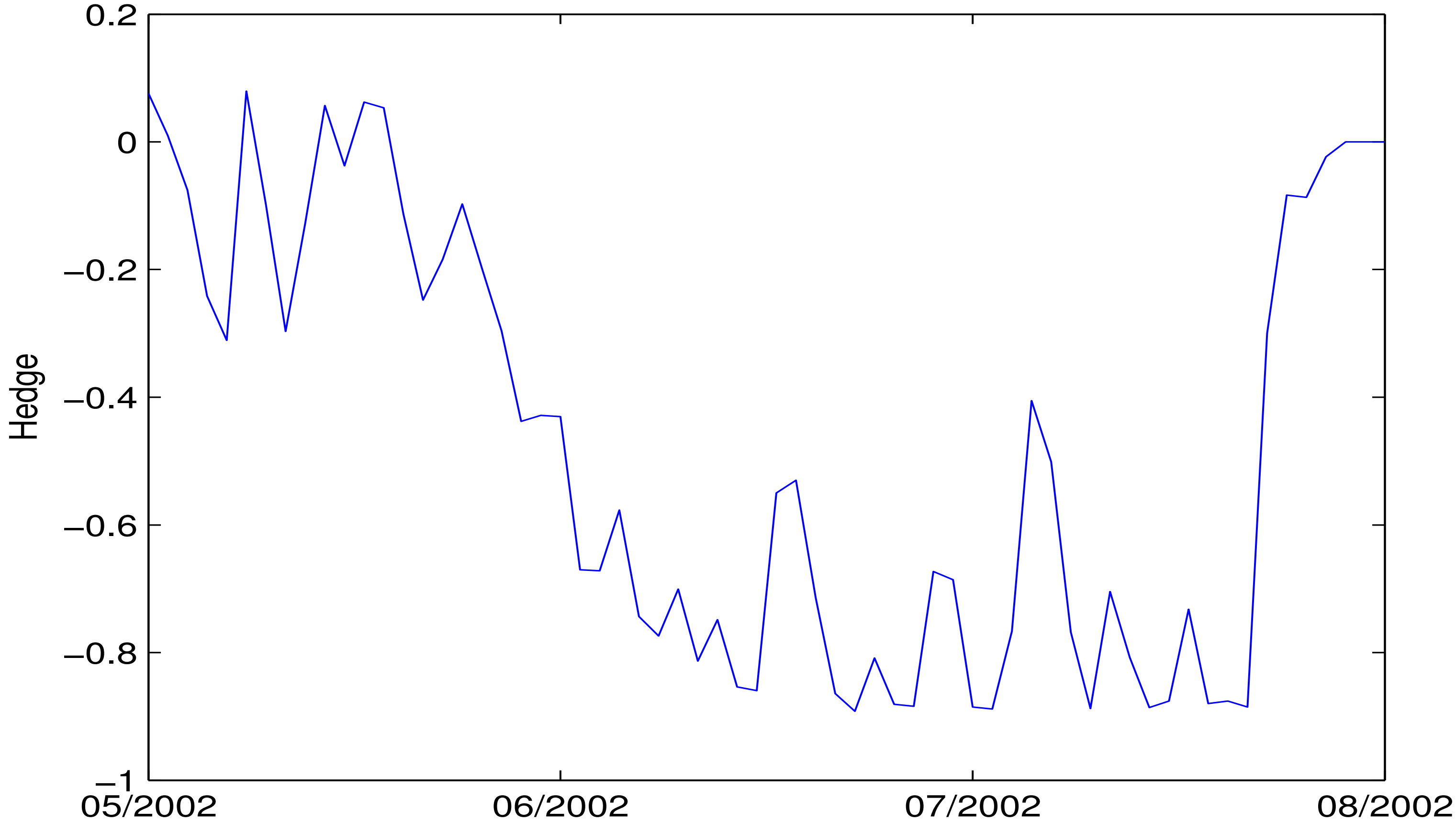
The corresponding hedge.



S&P 500 starting in May 2002 (worst 3 month drop in the history).



DD (blue), MDD (black), MDD Value (green), Replicating Portfolio (red)
3M MDD contract on S&P 500 starting in May 2002.



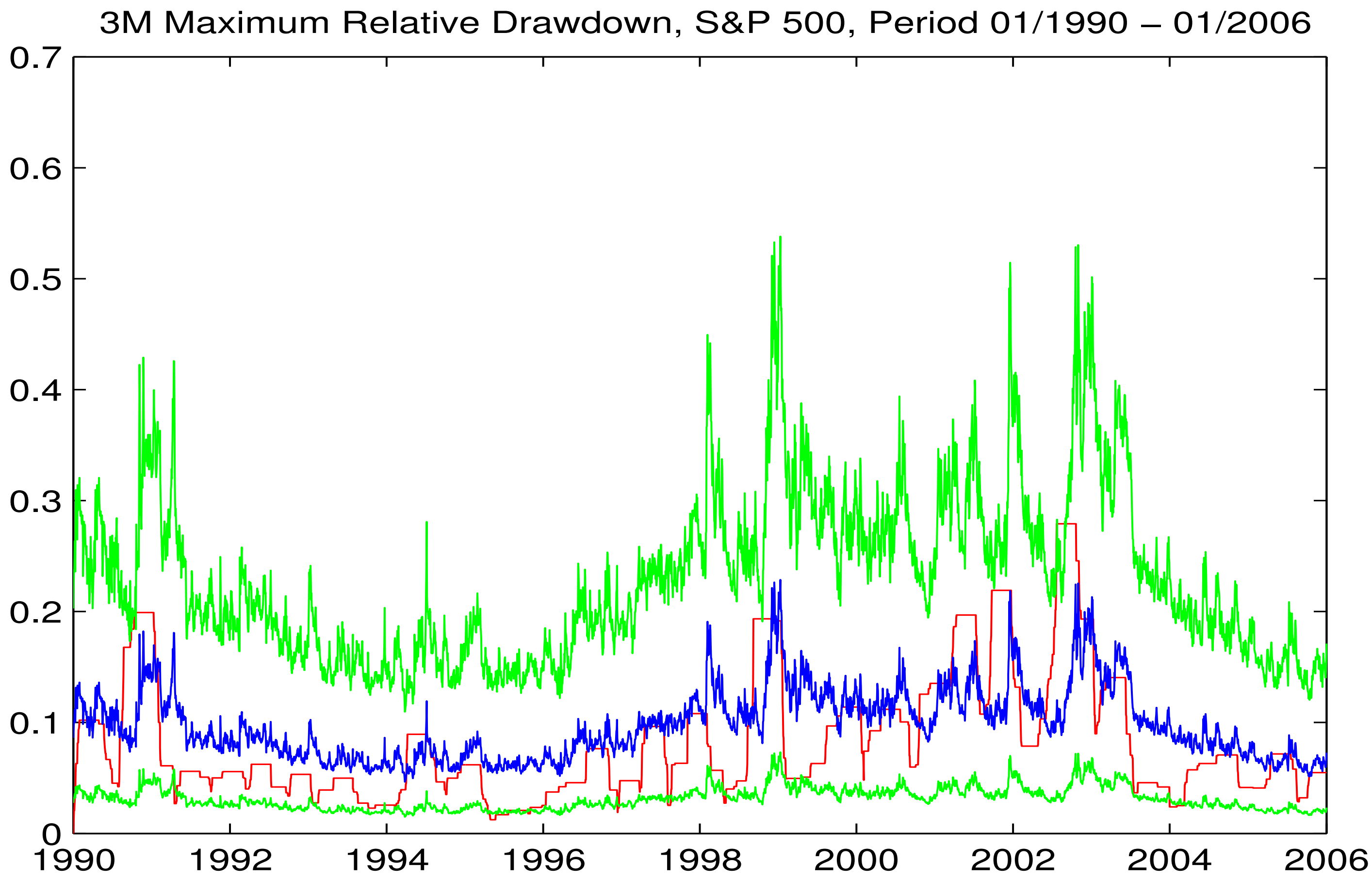
The corresponding hedge.

Severity of the Market Crashes

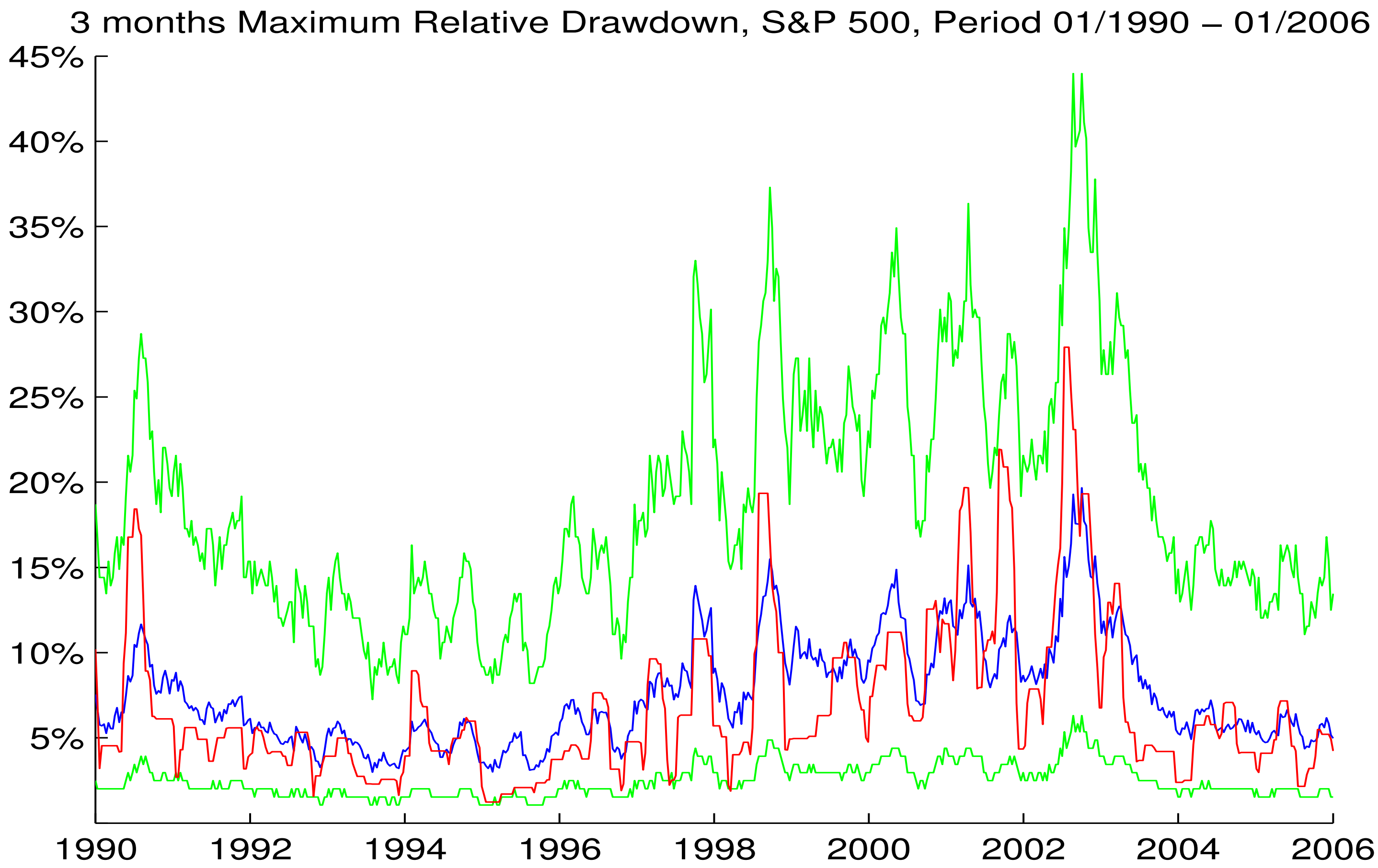
We can measure the severity of the market crash by giving the odds of the corresponding event. The odds are objective if we look at the (super)replicating costs.

This might seem counterintuitive – the worst crashes are the cheapest to replicate!!! This is great for portfolio insurance, one can adopt cheap hedges for events with odds 1:100, 1:1000, 1:1000000, etc.

Notice that the current volatility serves as an estimator of MDD. High volatility means high expectations for MDD and vice versa.



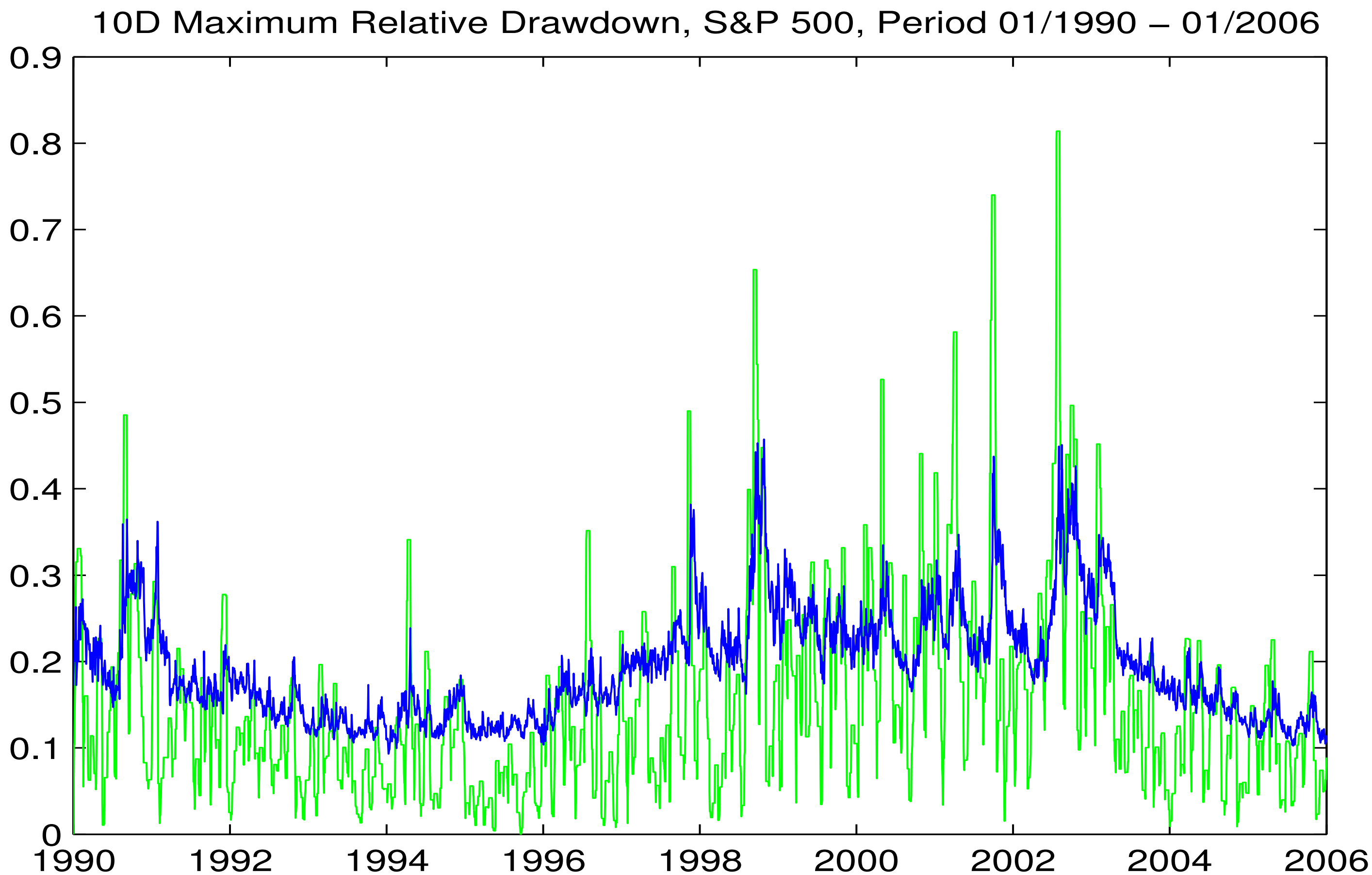
MDD (percentage) taken over 3M interval, S&P500, 01/1990 to 12/2005, the expected MRDD and the confidence intervals (ex ante).



MDD (percentage) taken over 3M interval, S&P500, 01/1990 to 12/2005, the expected MRDD and the confidence intervals (ex post).

Impact on Volatility

Maximum Drawdown is also a good predictor of volatility (similar to daily returns – GARCH models). However, MDD can also capture negative cummulative effect of small, but same sign consecutive daily returns.



5*MDD (percentage, in green), 10D interval,
S&P500 index, 01/1990 to 12/2005, and volatility index (blue).