Math151c, Spring 2008

1. Let $0 \to R \to F \to G \to 0$ be a short exact sequence of groups, and construct an associated fibration of spaces $K(R,1) \to K(F,1) \to K(G,1)$. Calculate the $E^2_{p,0}$ and the $E^2_{0,1}$ terms in this sequence, in terms of the homology of $K(G,1)$, the homology of $K(R,1)$, and the action of G on R by conjugation. Recall that the homology of a $K(G,1)$ is called *group homology*, and denoted $H_*(G)$. Use the spectral sequence of a fibration to obtain an exact sequence

$$H_2(F) \to H_2(G) \to H_1(R)_G \to H_1(F) \to H_1(G) \to 0$$

where $H_1(R)_G$ denotes the quotient of $H_1(R)$ by the (conjugation) action of G.

Now suppose F is free, and derive Hopf's formula for 2-dimensional group homology

$$H_2(G) = (R \cap [F,F])/[F,R]$$