LINEAR ALGEBRA, WINTER 2018, PRACTICE MIDTERM

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This practice midterm exam was posted online on Tuesday, February 5, and is due never, because it's not a real exam. When a problem asks you to give an example of something you must also explain why the example you give satisfies the demands of the problem.

Problem 1. Give two examples of vector space over \mathbb{R} — one that doesn't have a finite basis, and one that does.

Problem 2. Give an example of a linear transformation between two vector spaces of dimension 3 for which the image has dimension 2 and the kernel has dimension 1.

Problem 3. Let's let \mathbb{F} denote the finite field with two elements 0 and 1. Remember we can think of functions with values in \mathbb{F} as Boolean variables, by interpreting 0 as 'false' and 1 as 'true'. If A and B are Boolean variables, then A AND B returns true if A and B are both true, and false otherwise. Likewise, A XOR B returns true if either A or B are true but not both ('exclusive or'). Think of AND and XOR as functions from \mathbb{F}^2 to \mathbb{F}^1 . Are they linear?

Problem 4. Let Γ be the directed graph in Figure 1. How many loops in Γ are there of length 1000?



FIGURE 1. The directed graph Γ

Interpret the answer in terms of the trace of a certain 2×2 matrix.

Problem 5. Find all solutions to the following system of linear equations

$$3x_1 + 4x_2 - 7x_4 = 6$$

$$2x_2 + 3x_3 = 7$$

$$9x_1 + 9x_2 + 9x_3 = 9$$

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